Do payment systems matter: A new look*

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Abstract

In this paper, we consider two alternative pure payments systems—the trade of goods for goods, or barter, and trade using intrinsically valueless fiat money. Here, the term payment system refers to the method of executing mutually beneficial trades, and 'pure' means that each method of exchange is considered exclusively. Each payment system is examined in an economy with location-specific commodities, and households consist of vendor-shopper pairs. The household's decision problem includes a distance-related transaction cost; that is, the cost of trading with anyone from another location increases as the distance from the home location increases. We then ask, is the equilibrium set of consumption goods—and the quantity of each type—invariant to whether the vendor or the shopper pays the transaction cost? The answer is that in economies with monetary settlements, invariance fails.

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1 Introduction

In this paper, we consider two alternative pure payments systems—the trade of goods for goods, or barter, and trade using intrinsically valueless fiat money. Here, the term payment system refers to the method of executing mutually beneficial trades, and 'pure' means that each method of exchange is considered exclusively. Each payment system is examined in an economy with location-specific commodities, and households consist of vendor-shopper pairs.[1] The household's decision problem includes a distance-related transaction cost; that is, the cost of trading with anyone from another location increases as the distance from the home location increases.[2] We then ask, does it matter who pays the transaction cost? That is, is the equilibrium set of consumption goods—and the quantity of each type—invariant to whether the vendor or the shopper pays the cost to allow exchange?

The party that pays, in our sense, is more precisely characterized as the party that chooses a set of locations to exchange with (incurring a resource cost in the process). When vendors pay the cost, the households' shoppers take as given the set of locations open to trade with them. Conversely, when shoppers pay, the households' vendors take as given the set of locations from which shoppers will be visiting their home locations. Conditional on

[1]See Lucas and Stokey (1987) for an example of this type of household structure in a production economy. The idea of using spatial separation as a means of capturing transaction costs goes back at least as far as Townsend (1980).

[2]The appeal to transactions costs has a long tradition in monetary economics. See Baumol (1952) and Tobin (1956) for early examples. Saving (1971) later developed the shopping-time model in which fiat money is valued because it reduces the time lost executing trades. Later, Schreft (1992) presented the idea of distance-related transaction costs to analyze the use of cash and credit as means of payment in an overlapping generations model.

the payment system, does it matter for equilibria whether the households' vendors or shoppers pay, in this sense?

The answer is that the equilibria are invariant to who pays in the barter economy, but not in the monetary economy. In the monetary economy the equilibrium set of consumption goods chosen when the shopper pays Pareto dominates the equilibrium set of consumption goods when the vendor pays. The implication is that there is a kind of coordination failure that is present when the payment system involves monetary settlements. With money, we learn that the roles within the household are specialized in a way that does not occur when settlements involve exchanging goods. To be clear, the vendor specializes in acquiring money—generalized purchasing power—while taking the locations that the shopper trades with as given. The shopper specializes in acquiring goods—acting on a preference for variety—while taking the locations that the vendor trades with as given. In other words, each member of the household is solving a different problem when money is the means of settlement. When viewed this way, the main result is really not surprising at all. There are lots of papers that have examined how money fosters specialization that is welfare improving. To our knowledge, no one has characterized how specialization that exists in economies with monetary settlements can result in this kind of coordination problem.

The model economy is a modified version of Cole and Stockman (1992). At a fundamental level, the distance-related transaction cost creates a trade-off between variety and quantity. In related papers, variety has been considered in model economies to study production specialization. In Cole-Stockman, a person's taste for variety is embodied in a trade-off between self-production and trade with other agents. It is costly to produce a wider variety of different goods. Cole and Stockman show that in a monetary equilibrium, the measure of goods that are self-produced declines, thereby expanding the measure of goods acquired through trade. Money reduces the transaction costs associated with trade. Since each location produces a smaller measure of goods than they consume, Cole and Stockman conclude that valued fiat money promotes greater specialization. Camera et al. (2003) similarly define specialization as the set of goods that an agent produces. In a search model, Camera *et al.* consider a barter economy, deriving conditions in which specializing in producing one good reduces welfare.[3] In a monetary economy, fiat money is welfare improving by inducing production specialization. The idea of both of these papers is to demonstrate how fiat money can lower transaction costs across the variety of goods, induce people to specialize in production in which they have a comparative advantage, and thus expand the production possibility set compared with the one that exists in a barter equilibrium.

The remainder of the paper is organized as follows. Section 2 describes the general structure of the economic environment in terms of household preferences, locations, endowments and transaction cost functions. In section 3, we consider the case of barter as the payment system and describe equilibrium outcomes for two economies—one in which the vendor pays the transaction cost of any exchange and one in which the shopper pays the cost. Similarly, section 4 derives the equilibrium outcomes for the monetary economy, comparing outcomes under 'vendor pays' and 'shopper pays' rules. We offer a brief summary in Section 5. An appendix contains proofs of some claims made in the text.

[3] Product specialization is also embedded in the random matching models. See Kiyotaki and Wright (1993) and Lagos and Wright (2005).

2 The environment

The physical environment we will describe can be interpreted as a group of households, each living at a specific stretch of beach on an atoll, which we idealize as a circle.

More formally, there is a large, finite number of infinitely-lived households living at distinct locations along a circle. Time is discrete and indexed by $t = 0, 1, 2, ...$ For our purposes, let there be $N > 2$ locations equally spaced on the circle; by implication, the circle has circumference N. For symmetry, we assume that each location is populated by a large number of identical households. Each household consists of a vendor and a shopper. The vendor stays at the home location, trading with visitors from other locations on the atoll. Meanwhile, the shopper visits locations along the atoll to purchase goods for the household.[4] Thus, N is also the number of household types. The households at each location are endowed with units of a nonstorable, location-specific good, so that there are also N types of commodities at each date.

Trade takes place as agents from each household move around the atoll to visit the locations of other households. Let $i \in \{0, 1, 2, ..., N-1\} \equiv \mathcal{N}$

[4] The interpretation of a two-person household was developed in Lucas and Stokey (1987) where there was a worker and a shopper. We could drop the vendor-shopper pair since it is equivalent to interpret the physical environment as one in which there are a large number of agents living at each location, with each agent operating a vending machine at their home location (à la Cole-Stockman). To get at our two types of experiments, consider the vending machine as having two versions. In one version, the home-location agent prepays the transaction cost and chooses the visiting locations with which the vending machine will execute trades. Alternatively, the vending machine can require the purchaser to pay the transaction cost.

index locations on the atoll; hence, i indexes both the locations from which the household hails and the goods.[5]

We assume that consumers do not derive utility from consuming their home-location good. On the other hand, we assume that households at each location do derive utility from the goods at *all* other locations, and that households have identical preferences defined over the full range of goods (modulo the home good). There is thus no double coincidence of wants problem in our economy.

At the start of each period, the household's shopper travels to other locations on the circle to purchase goods, either with units of the home good or cash, depending upon the payment system under consideration. Meanwhile, the vendor remains at the home location to transact with the shoppers of households from other locations.[6]

Travel by the shopper half of the household is restricted to one direction; after visiting as many locations as he or she chooses to visit in this direction, the shopper returns home by the same route. For concreteness, assume the locations are arranged in ascending order clockwise around the circle, with $i = 0$ at the top, and that the direction of travel by shoppers is also clockwise.

In order for exchange to take place between households at locations i and j, a direct resource cost must be borne by one of the parties to the

[5] Throughout our analysis, we use discrete locations with positive measure, following Freeman (1996). In several places, we provide some intuition by considering the results using a continuum of locations along a unit circle. The integer-programming approach, however, is used throughout so that the limiting result has positive measure.

[6] As in the Lucas-Stokey framework, the key feature is that the pair cannot perfectly coordinate their activities to overcome trading frictions. In the monetary economy of section 4 below, the pair is similar as well to the 'vending machines' and shoppers of Cole and Stockman.

trade. More specifically, the cost is paid in units of the endowment good of whichever party is assumed to bear the cost in a particular environment. The cost is independent of the quantity of goods traded and is increasing in the distance—measured along the circumference of the circle—between locations i and j. In short, the cost is a fixed transaction cost at each location and is increasing in the distance between the transacting parties' home locations.

In order to keep the model simple, so as to focus on the role of the transaction cost, we assume that the number of households of each type is sufficiently large that each household acts as a price-taker at all locations which its shopper visits and with all shoppers visiting the household's location. With this interpretation in mind, we proceed to lay out the model in more precise detail.

The structure of preferences is identical across households, and the preferences of each household treat all goods symmetrically. The momentary utility function for each household type $i \in \mathcal{N}$ is represented by

$$U_t = \left[\sum_{j \in \mathcal{N} \setminus \{i\}} c_t(j)^\alpha \right]^{1/\alpha} \tag{1}$$

where $0 < \alpha < 1$, $\mathcal{N} \setminus \{i\}$ is the relative complement of $\{i\}$ in $\mathcal{N}$, and $c_t : \mathcal{N} \setminus \{i\} \to R_+$ is the consumption 'bundle' at date t. Each household seeks to maximize the discounted sum

$$\sum_{t=0}^{\infty} \beta^t U_t$$

where $0 < \beta < 1$.

Identical preferences makes the analysis substantially more tractable. For one thing, given the further assumptions we make below on transactions costs, we can conduct our analysis for a representative household—the household at location 0—without loss of generality.

Each household i is endowed with $e_t(i) > 0$ units of commodity i at each date t. The endowment goods are perishable. We will assume that endowment levels are identical across households and across time; that is $e_t(i) = e$ for all i and t.

We have not yet developed a specific role for spatial separation. Here, its force derives from the transaction cost's dependence on distance. We will consider environments where this cost is borne by either the vendor or the shopper in a given transaction. In the 'shopper-pays' environment, a shopper who travels from the home location to a location k units away—*e.g.*, from location 0 to location k—pays a cost $a(k)$ before trade can take place. In the 'vendor-pays' environment, the vendor at any location who wishes to trade with a shopper coming from a location k units away—*e.g.*, from location $N - k$ to 0—must incur the cost $a(k)$ before trade can take place.

A trading range is defined as the set of locations with which the household member will seek to trade, when the choice is theirs to make.[7] In the shopper-pays case, the shopper will choose a range of locations for which he is willing to pay the transaction cost in order to trade with the vendors at those locations. Similarly, the trading range in the vendor-pays case consists of those locations for which the vendor is willing to pay the transaction cost in order to trade with shoppers visiting from those locations.

As we develop the analysis, we will further distinguish the shopper-pays and vendor-pays cases in the context of the household's problem. For now,

[7]Our language here anticipates the result that the set will consist of adjacent locations—it will never be optimal to skip a location to trade with another farther away.

it is sufficient to note that the transaction cost represents resources used up and subtracted from the payer's endowment. Thus, for example, in the shopper-pays environment, if the shopper from a household at location 0 visits locations 1 through k, the household's endowment net of transactions costs is

$$e - \sum_{j=1}^{k} a(j).$$

Note that this also the net endowment of the household at location 0 in the vendor-pays environment if the household trades with shoppers coming from locations $N - k$ through $N - 1$.

We make some assumptions on the transaction cost function $a(\cdot)$ in order to guarantee nontrivial equilibria. In particular, we assume that $a(k)$ is increasing in k and that there is a $\hat{k} \in \mathcal{N}$ such that $0 < \hat{k} < N - 1$ with $e > \sum_{i=1}^{k} a(i)$ for $k \leq \hat{k}$ and $\sum_{i=1}^{k} a(i) > e$ for $k > \hat{k}$. In words, it's feasible for a shopper or vendor to transact with *some* locations, but too costly to transact with *all* locations.

We can show there are gains from trade under our assumptions on tastes and technology. Given our assumptions on the transaction cost function a, it's straightforward that there exist $c > 0$ and $k > 1$ such that

$$e - \sum_{j=1}^{k} a(j) \geq 2kc. \tag{2}$$

From the inequality (2) it follows that a feasible allocation exists in which each household's shopper visits the first k locations in the direction of travel from the home location, each household's vendor trades with visitors from the k locations lying in the counterclockwise direction, and each household consumes c units of every good from these $2k$ locations.[8] The utility each

[8]The transaction cost associated with these trades at the k different locations is

household receives from this allocation is

$$[kc^{\alpha} + kc^{\alpha}]^{1/\alpha} = [2kc^{\alpha}]^{1/\alpha} > 0.$$

With this basic environment in place, we now investigate whether it matters if the shopper or the vendor pays the transaction costs across three pure payments systems. The assumption of which party bears the transaction cost would seem to be innocuous in terms of affecting equilibrium outcomes under a given payment system. In the next section, we present a case in which the equilibrium is in fact identical regardless of whether the shopper or the vendor pays the transactions fee.

3 Payment System I: Barter

In this section, we consider trading environments in which the vendor and shopper exchange units of their endowment goods—*i.e.*, barter economies. Given our assumption that households are price-takers, the equilibria we focus on are competitive equilibria. And, given symmetric transaction costs and preferences—and, moreover, preferences which treat all goods identically—it is natural to focus on competitive equilibria which are *symmetric*. By symmetric equilibria, we mean equilibria in which:

1. All households trade with households from k adjacent locations lying in both directions from the home location.

2. Households' consumption bundles are identical.

3. For any i and j, the relative price of the good at location i in terms of the good at location j depends only on the distance between i and j.

$\sum_{i=1}^{k} a(i)$ for each household; the inequality (2) then states that the endowments net of those transaction costs are sufficient to consume c units of each good from $2k$ locations.

Moreover, as we discuss further below (and prove in Appendix B), a necessary feature of symmetric equilibria is that *all relative prices are unity.* That is, if we restrict attention to equilibria where households make identical choices and relative prices depend only on distance—which are natural assumptions given the environment—it follows that we may further restrict attention to equilibria where all relative prices are unity. Thus, we will focus on equilibria obeying properties 1, 2 and

$3'$. For any i and j, the relative price of the good at location i in terms of the good at location j is 1.

Note that from $3'$, it will also follow that household consumption bundles are constant across locations—$c_t(i) = c_t(j) = c_t$ for all i and j.

Because of the symmetry of preferences and transaction costs, we can discuss the problem from the perspective of a representative household located at $i = 0$, without loss of generality. To elaborate the barter economy in concrete terms, we first consider the case where the vendor pays the transaction cost associated with any exchange. Later, we show how the expressions characterizing equilibria change (or not) when shoppers bear the transaction cost.

3.1 Barter equilibria in the vendor-pays environment

When the vendor is responsible for the transaction fee, each household will choose a set of visiting shoppers with whom its vendor is willing to trade—*i.e.*, a set of locations, lying in the counterclockwise direction from the home location, from which the household will accept goods in exchange for units of the home endowment. Consequently, each household will take as given the set of locations, lying in the clockwise direction from the home location, which are 'open' to its own shopper—*i.e.*, those locations around the atoll where other households have incurred the fixed cost to trade the goods which the shopper carries from the home location.

For the household located at $i = 0$, call the set of locations visited by the shopper S_t and the set of locations from which the vendor accepts visitors S'_t. Because the transaction cost increases with distance, and all goods are treated symmetrically in households' preferences, we may assume without loss of generality that the sets S_t and S'_t are each 'connected' in the sense that S_t consists of all locations 1 through k_t for some k_t, and S'_t consists of all locations $N-1$ through $N-k'_t$ for some k'_t. Vendors and shoppers will never skip over a location to trade with one that is more distant; rather, they trade incrementally, choosing a set of adjacent locations and balancing the desire to eat each of the $N-1$ non-home, differentiated goods against the transactions costs.[9]

It is also clear that, because of the increasing transaction cost, taking as given S_t, the household will always choose S'_t such that $S_t \cap S'_t = \varnothing$—it would be inefficient for the household to pay the fixed cost to include a location in S'_t if that location is already open to the household's shopper in S_t.

We let $A\left(S'_t\right)$ denote the total cost of trading goods from locations in S'_t—*i.e.*,

$$A\left(S'_t\right) = \sum_{i=1}^{k'_t} a\left(i\right).$$

Now, suppose that all relative prices are unity, as they would be in a symmetric equilibrium; we establish this property in Appendix B. Under

[9]We give a proof of this 'connectedness' in Appendix A.

this assumption, the household's budget constraint can be written as

$$e - A(S_t') \geq \sum_{j \in S_t} c(j) + \sum_{j \in S_t'} c'(j) \tag{3}$$

We will construct an allocation where each household is maximizing its utility subject to its budget constraint, household choices are symmetric, and markets clear.

Since the good is nonstorable, and exchange of goods for goods is the only means of trade, each household's lifetime utility-maximization problem amounts to a static problem of maximizing momentary utility at each date. Given the household's preferences (1) and the budget constraint (3), the household's optimal consumption bundle will obey $c_t(i) = c_t(j)$ for all $i, j \in S_t$ and $c_t(i) = c_t(j)$ for all $i, j \in S_t'$. Let c_t denote the constant level of consumption on the set S_t and c_t' the constant level of consumption on S_t'. The household's budget constraint can then be written as

$$e - A\left(S_t'\right) \geq c_t \left|S_t\right| + c_t' \left|S_t'\right|, \tag{4}$$

where $|S_t|$ and $|S_t'|$ are integers representing the numbers of locations in the sets S_t and S_t', respectively.

Using this notation, the household's momentary utility from consuming c_t on S_t and c_t' on S_t' can be written as

$$U_t = \left((c_t)^\alpha \left|S_t\right| + \left(c_t'\right)^\alpha \left|S_t'\right|\right)^{1/\alpha}. \tag{5}$$

Taking as given S_t, the household chooses c_t, c_t' and S_t' to maximize utility (5) subject to the budget constraint (4).

Because of the fixed cost, this problem is not convex, but may be approached as follows. Temporarily taking S_t' as well as S_t as given, we can calculate optimal choices of c_t and c_t'. This gives rise to an indirect utility function in terms of S_t' and S_t, and we can find the optimal choice of S_t' given S_t. Finally, we impose symmetry—$S_t' = S_t$—to arrive at a characterization of a symmetric competitive equilibrium.

The friction is embodied in the formulation of the household's two different economic problems. The indirect utility function solves for S_t', taking S_t as given or vice versa. In the case in which the vendor pays, consider the vendor solves the problem, communicates costlessly to the shopper, who then knows how many locations are willing to trade with the shopper. In this illustration, when the vendor solves for S_t' it simultaneously solves for S_t. By taking S_t as given, we are implicitly treating the shopper as being oblivious to what the vendor is going to choose. We will focus on equilibrium allocations with this type of friction present in all of our analyses.

It is straightforward from the form of (4) and (5) that, given S_t, and for a given choice of S_t', the optimal choices of c_t and c_t' must obey

$$c_t = c_t' = \frac{e - A\left(S_t\right)}{|S_t| + |S_t'|}.$$

The implication is that consumption levels on the two sets are equated, and the budget constraint is satisfied with equality. The household's momentary utility can then be written in terms of S_t and S_t' as

$$U_t = \left(e - A\left(S_t'\right)\right)\left(|S_t| + \left|S_t'\right|\right)^{\frac{1-\alpha}{\alpha}}. \tag{6}$$

From (6), a household's utility is increasing in the cardinality of the set S_t' and decreasing in the transaction cost associated with S_t'. It follows that if S_t' is an optimal choice, it must have the smallest transaction cost $A(S)$ over all sets S with cardinality $|S_t'|$, from which it becomes clear that the optimal choice does indeed have the form $\{N-1, N-2, ..., N-k_t'\}$. If S_t also has the 'connected' form $\{1, 2, ..., k_t\}$, then $|S_t'| = k_t'$, $|S_t| = k_t$, and the optimal choice of S_t' reduces to the choice of the integer k_t' that solves

$$\max_h \left(e - \sum_{i=1}^{h} a(i) \right) (k_t + h)^{\frac{1-\alpha}{\alpha}}.$$

This is an integer-programming problem, the solution of which can be characterized by a set of inequalities. For our purposes in this paper, having very tight characterizations of equilibria is inessential for showing how equilibria either differ or do not differ across different environments. It is sufficient to note that a symmetric competitive equilibrium in the current environment, if one exists, is characterized by

$$k_t = \arg\max_h \left(e - \sum_{i=1}^{h} a(i) \right) (k_t + h)^{\frac{1-\alpha}{\alpha}}. \tag{7}$$

The critical feature of (7) is that the vendor chooses the distance for which they are willing to pay the transaction cost, taking the locations visited by the shopper as given. In doing so, the marginal cost of accepting a shopper from the next farthest location is equated with the marginal gain from consuming an additional variety. In equilibrium, consumption by each household from each of the $2k_t$ locations is given by

$$c_t = \frac{e - \sum_{i=1}^{k_t} a(i)}{2k_t}. \tag{8}$$

Note that the number of goods each household consumes is $2k_t$.[10]

[10]Heuristically, one can get a feel for the equilibrium by imagining, for a moment, that there are a continuum of locations, in which case the household's maximization would give the following first-order condition:

$$a(k_t') \left[k_t + k_t' \right] = \frac{1-\alpha}{\alpha} \left(e - \int_0^{k_t'} a(i)\, di \right).$$

In a symmetric equilibrium $k_t = k_t'$, and the common value k_t would be characterized by

$$a(k_t)\, k_t = \frac{1-\alpha}{2\alpha} \left(e - \int_0^{k_t} a(i)\, di \right).$$

3.2 Barter equilibria in the shopper-pays environment

Now suppose that it is the shopper who pays the fixed cost associated with any exchange, hence chooses the set of vendors with whom the household will trade. In this case, the typical household takes as given a set S'_t of shoppers from other locations who will be visiting the home location, and chooses a set S_t of locations which its shopper will visit. Again assume that all relative prices are unity, from which it follows that the household sets $S_t \cap S'_t = \varnothing$ and chooses constant consumption levels c_t and c'_t on the two sets. The budget constraint again takes the form

$$e - A\left(S_t\right) \geq c_t \left|S_t\right| + c'_t \left|S'_t\right|,$$

where $A(S_t)$ is the sum of the transactions costs which the household incurs from shopping at the locations in S_t. By way of comparison with the previous environment, note that the household's cost of visiting a set $\{1, 2, ..., k_t\}$ of locations in this environment would be identical to its cost of transacting with shoppers visiting from locations $\{N-1, N-1, ..., N-k_t\}$ in the previous environment.

The household's momentary utility is again given by

$$U_t = \left(\left(c_t\right)^\alpha \left|S_t\right| + \left(c'_t\right)^\alpha \left|S'_t\right|\right)^{1/\alpha}.$$

It's immediate that we again have $c_t = c'_t$ at an optimum. An argument similar to that above shows that S_t takes the form $\{1, 2, ..., k_t\}$, and that the

Note that existence of a k_t satisfying the last expression is essentially immediate from the assumptions that $a(\cdot)$ is nonnegative, continuous, and increasing, and such that $\int_0^k a(i)di > e$ for all k greater than some $\hat{k}$; the left-hand side is then increasing from a value of zero at $k = 0$, while the right-hand side is decreasing from a positive value at $k = 0$ to negative values for $k > \hat{k}$.

optimal choice of k_t, given $S_t' = \{N-1, N-2, ..., N-k_t'\}$, is the solution to

$$\max_h \left(e - \sum_{i=1}^{h} a(i) \right) \left(h + k_t'\right)^{\frac{1-\alpha}{\alpha}}.$$

Consequently, a symmetric equilibrium is again characterized by

$$k_t = \arg\max_h \left(e - \sum_{i=1}^{h} a(i) \right) (h + k_t)^{\frac{1-\alpha}{\alpha}}, \tag{9}$$

and

$$c_t = \frac{e - \sum_{i=1}^{k_i} a(i)}{2k_t}. \tag{10}$$

Note that the expressions in (7) and (8) are identical to equations (9) and (10). Since these equations completely characterize equilibria in the two environments, the analysis shows that the equilibrium outcomes are identical for the two versions of this barter model economy. More specifically, the representative household maximizes utility in equilibrium by choosing the same consumption bundle—that is, the same level of consumption from each location and the same range of locations with which to trade. Hence, the vendor-pays economy is equivalent to the shopper-pays economy. This is a general feature of economies in which exchange is a trade of endowment goods for endowment goods.[11]

3.3 Guaranteeing existence of symmetric equilibria

Symmetric equilibria in the barter economy, when they exist, are identical regardless of whether the shopper or vendor pays the transaction cost—but do we know symmetric equilibria exist? Existence is straightforward

[11] Suppose that the transactions costs are borne according to the following rule: the seller pays $\theta a(i)$ and the shopper pays $(1-\theta)a(i)$, for $0 \leq \theta \leq 1$. It is fairly straightforward to show that the results, in terms of range of goods consumed (k) and the quantity of each good consumed (c) would be identical for any θ.

to show with a continuum of locations, assuming only that the transaction cost function $a(\cdot)$ is nonnegative, continuous, increasing, and such that $\int_0^k a(i)di > e$ for all k greater than some $\hat{k}$—see footnote 10 above. With a discrete set of locations, which we employ primarily for its simplicity in other regards, giving minimal assumptions on $a(\cdot)$ that guarantee existence is more difficult. Hence, we will simply assume that $a(\cdot)$, e, and α—in addition to the assumptions already made—are such that there exists a k with

$$\left(e - \sum_{i=1}^{k} a\left(i\right)\right)(2k)^{\frac{1-\alpha}{\alpha}} \geq \left(e - \sum_{i=1}^{h} a\left(i\right)\right)(h+k)^{\frac{1-\alpha}{\alpha}} \tag{11}$$

for all h. That is, we assume the fixed-point problem implicit in (7)—or equivalently (9)—has a solution.[12]

We have in place all the pieces to verify that symmetric competitive equilibria exist, and have the properties described above—namely, that all relative prices are one and that allocations are invariant to the identity of party bearing the transaction cost. In sum, we can show:

Proposition 1. *In the barter economy, there exists a symmetric competitive equilibrium in which the location-specific goods trade at a relative price equal to one and the range of locations and the quantity are represented equivalently by equations* (7) *and* (8) *or equations* (9) *and* (10)*.*

Proof. The derivations of (7)–(8) and (9)–(10) are given above, under the assumption that all relative prices are unity. That there exists a k_t that solves (7) and (9) follows immediately from the assumption that $a(\cdot)$ obeys

[12] Given the complex nature of this joint assumption on $a(\cdot)$, e, and α, it behooves us to show that such $a(\cdot)$, e, and α exist. Suppose there are four locations on the circle, $i = 0, 1, 2, 3$, let $e = 1$, $\alpha = 1/2$, and $a(\{0, 1, 2, 3\}) = \{0, 1/4, 1/2, 3/4\}$. It's easily verified that a symmetric equilibrium exists with $k = 1$ and $c = 3/8$.

(11). All that remains to be shown, then, is that relative prices are in fact unity in a symmetric equilibrium. We prove that in Appendix B. □

4 Payment System II: Fiat money

In this section, we consider an environment in which there is a store of value, fiat money, which is the sole means of exchange. We are not trying to explain why money is superior to barter. In our framework—where barter is not subject to double coincidence of wants problems or search frictions—money may be a superior medium of exchange owing to lower distance-related transactions costs. That is, the transaction cost function $a(\,\cdot\,)$ may be uniformly lower with money as the means of exchange.[13] Our focus is whether the monetary equilibria under vendor-pays or shopper-pays rules display the same invariance as they do in the barter economy.

As usual, we assume that fiat money is intrinsically useless and noncounterfeitable. Let the stock of money be constant over time. Trade takes place as before, with shoppers from each household moving clockwise around the atoll. In this economy however, all trades take the form of shoppers offering cash to vendors in exchange for goods. Note that in this environment, a household only consumes goods lying in the shopper's direction of travel from the home location.

As in exchanges in which the endowment goods are used as payment, we assume that there is a fixed cost that is related to the distance between

[13]We use the same notation for the transactions cost throughout, and make similar assumptions regarding its nonnegativity and dependence on distance, but we do not assume that the function $a(\,\cdot\,)$ here is identical to the $a(\,\cdot\,)$ from section 3. Rather, the notation facilitates comparing equilibrium allocations across the two environments.

two potential traders. We consider the same two cases: either the shopper or vendor pays a fixed fee to trade with persons that live j locations away.

In this economy, the separation of the shopper-vendor pair at the start of each period presents a timing issue. The vendor must offer the home-good for cash while the shopper uses the household's previously accumulated cash balances to finance the pair's current-period consumption. At the end of the period, the vendor gives the shopper the proceeds from this period's sales to finance next-period's consumption. *De facto,* a cash-in-advance condition arises.

Analogous to our notation of the last section, let S_t and S_t' denote, respectively, the set of locations to which the shopper will carry cash to exchange for goods and the set of locations from which other households' shoppers will visit bearing cash to exchange for the home endowment good. If the shopper is responsible for the cost of verifying that the goods received satisfy the conditions for trade, the household chooses the set S_t of locations to visit, and takes as given the set S_t' of visitors. In this case, for the household at location zero, trading at a set of locations S_t incurs a cost of $A\left(S_t\right) = \sum_{i \in S_t} a\left(i\right)$,where $a\left(i\right)$ is again increasing in i, with $\sum_{i=1}^{k} a\left(i\right) < e$ for k small, and $\sum_{i=1}^{k} a\left(i\right) > e$ for k large. Conversely, if the vendor pays the distance-related fixed cost associated with any potential trading partner, the household chooses the set S_t' of shoppers from whom the household's vendor will accept cash in exchange for the home good and takes as given the set S_t of markets to which the shopper carries money. In this case, the household would incur a cost $A\left(S_t'\right) = \sum_{i \in S_t'} a\left(i\right)$, which comes out of the pair's endowment of the home good. In either case, assuming that the *de facto* cash-in-advance constraint is binding, the household's money balances at the start of the next period will be the nominal value of the household's endowment, less transactions costs.

Without loss of generality, again suppose the relative prices of goods are equal to unity. The household starts the period with a quantity of real cash balances, denoted by m_t. Given the set S_t of markets to which the household carries cash, consumption on that set—which will be uniform given unit relative prices—obeys:

$$c_t |S_t| \leq m_t. \tag{12}$$

This is the household's cash-in-advance constraint: purchases of current consumption by the shopper must be financed with previously accumulated cash balances. Assuming that (12) binds, the household's real money balances in the subsequent period are given by either

$$m_{t+1} = \frac{p_t}{p_{t+1}} \left[e - A\left(S_t\right)\right], \tag{13}$$

or

$$m_{t+1} = \frac{p_t}{p_{t+1}} \left[e - A\left(S_t'\right)\right], \tag{14}$$

depending on whether the household incurs the transaction costs through shopping (13) or vending (14). Here, p_t is the price in units of cash of a unit of the home endowment at date t. The endowment net of the transaction cost—e less either $A(S_t)$ or $A(S_t')$—is sold by the vendor to shoppers from locations in S_t' in exchange for cash, yielding either $p_t [e - A(S_t)]$ or $p_t [e - A(S_t')]$ units of currency for the household. The real purchasing power of the household's currency next period is then either $p_t [e - A(S_t)] / p_{t+1}$ or $p_t [e - A(S_t')] / p_{t+1}$.

Substituting c_t from (12), as an equality, into the household's momentary utility function (1), gives the following expression for the household's within-period utility, in terms of S_t and m_t:

$$U_t = \left[m_t^{\alpha} \left|S_t\right|^{1-\alpha}\right]^{1/\alpha}. \tag{15}$$

We then can cast the household's lifetime utility-maximization problem as one of the following two dynamic programs, depending on whether we are in the shopper-pays or the vendor-pays environment:

$$v\left(m_t; \mathbf{z}_t\right) = \max_{S_t} \left\{ \left[m_t^{\alpha} \left|S_t\right|^{1-\alpha}\right]^{1/\alpha} + \beta v \left(\frac{p_t}{p_{t+1}} \left[e - A\left(S_t\right)\right]; \mathbf{z}_{t+1} \right) \right\} \tag{16}$$

or

$$v\left(m_t; \mathbf{z}_t\right) = \max_{S_t'} \left\{ \left[m_t^{\alpha} \left|S_t\right|^{1-\alpha}\right]^{1/\alpha} + \beta v \left(\frac{p_t}{p_{t+1}} \left[e - A\left(S_t'\right)\right]; \mathbf{z}_{t+1} \right) \right\} \tag{17}$$

The $\mathbf{z}_t$ in these Bellman equations denotes the vector of all exogenous variables which condition the household's decision at each date, in particular the price level p_t. The character of equilibria in the two environments hinges on the very different natures of the solutions to these two problems.

4.1 Monetary equilibria in the shopper-pays environment

Consider (16) first, which corresponds to the 'shopper pays' environment. Assuming S_t takes the form $S_t = \{1, 2, ..., k_t\}$—*i.e.*, an interval in the direction of travel from the home location to some k_t—the Bellman equation becomes

$$v\left(m_t; \mathbf{z}_t\right) = \max_{k_t} \left[m_t^{\alpha} k_t^{1-\alpha}\right]^{1/\alpha} + \beta v \left(\frac{p_t}{p_{t+1}} \left[e - \sum_{i=1}^{k_t} a\left(i\right)\right]; \mathbf{z}_{t+1} \right).$$

Note that changing k_t results in both costs and benefits to the household—the household's momentary utility is increasing in the range of locations visited by the shopper, but a greater range of locations comes at the cost of smaller real cash balances for next period. An optimal choice of k_t balances these effects. Of course, the maximization on the right-hand side of this Bellman equation is an integer-programming problem, as k_t is restricted to integer values. It would be straightforward to add enough additional structure to fully characterize a solution; however, as in our analysis of the barter economy, having very tight characterizations of equilibria is not important for demonstrating how, in broad terms, equilibria differ across different environments.

Even without explicitly solving this problem, we can draw some conclusions about the character of the solution. The most important feature to note is that there is no explicit dependence of the household's problem on S'_t, the set of visitors to the home location. This follows from the faceless nature of the household's monetary transactions—its endowment net of transactions costs is worth $p_t\left[e - A\left(S_t\right)\right]$ independent of the identity of the buyers who purchase it. This feature of transactions using money proves to be important for comparing the nature of equilibria in the shopper-pays versus vendor-pays environments.

Remark. *As in our analysis of the barter economy, further intuition can be gained by assuming for a moment, that locations are continuous, so that the problem is not integer-constrained. If the value function is differentiable, the first-order condition for the right-hand-side maximization is*

$$\frac{1-\alpha}{\alpha} m_t k_t^{\frac{1-\alpha}{\alpha}-1} = \beta v'\left(m_{t+1}; \mathbf{z}_{t+1}\right) \frac{p_t}{p_{t+1}} a\left(k_t\right)$$

while the envelope condition is

$$v'\left(m_t; \mathbf{z}_t\right) = k_t^{\frac{1-\alpha}{\alpha}}.$$

In a steady-state equilibrium, with m_t, k_t and p_t constant, these combine to *give:*

$$a(k)\,k = \frac{1-\alpha}{\alpha\beta} m$$

or

$$a(k)\,k = \frac{1-\alpha}{\alpha\beta}\left[e - \int_0^k a(i)\,di\right],$$

where the last equality follows from the fact that $m = e - \int_0^k a(i)\,di$ *when* p_t*,* m_t *and* k_t *are all constant. The household's steady state consumption (per location) is given by* $kc = m$*, or*

$$c = \left(e - \int_0^k a(i)\,di\right)/k.$$

4.2 Monetary equilibria in the vendor-pays environment

Now, consider (17), the dynamic program which the household faces in the vendor-pays environment. The key differences between the problems described by (16) and (17) are that in the latter, the $|S_t|$ entering the household's one-period reward—the set of locations which are open to the household's shopper—is taken as given, while the quantity of real balances the household takes into the subsequent period now depends on the household's choice of S_t', the set of locations from which the household will accept cash in exchange for the home endowment. That is, the range of goods available to the household's shopper depends on other households' decisions as to whether or not to incur the cost of transacting with the shopper, while the household's vendor makes a similar decision regarding transacting with other households' shoppers.

If S_t and S_t' are intervals of the form $\{1, \ldots, k_t\}$ and $\{N - k_t', \ldots, N-1\}$, this problem can be written as—

$$v(m_t; \mathbf{z}_t) = \max_{k_t'} \left\{ \left[m_t^{\alpha} k_t^{1-\alpha}\right]^{1/\alpha} + \beta v\left(\frac{p_t}{p_{t+1}}\left[e - \sum_{i=1}^{k_t'} a(i)\right]; \mathbf{z}_{t+1}\right)\right\}.$$

This problem has a simple solution—since k_t is given, and the household's next-period money balances are decreasing in k_t', the household chooses the smallest possible set on which to sell its endowment. That is, the household will set $k_t' = 1$, or $S_t' = \{N-1\}$, offering its endowment in exchange for cash only to shoppers from the nearest adjacent location.

The assumption of price-taking behavior means that the vendor can sell *any amount* of the home good at p_t dollars per unit on *any* S_t'. Given that the verification cost $A(S_t')$ is increasing in S_t'—and thus next-period's real balances are decreasing in S_t'—the best thing for the household to do is to sell $e - a(1)$ to the shopper from location $N-1$—*i.e.*, vend the whole endowment to the shopper from next door. In a symmetric equilibrium—with all households following this same logic—everyone exchanges with and consumes only the goods of the households at their nearest neighboring location.

This stark outcome highlights what it means for fiat money to serve as a generally acceptable medium of exchange. The problem seems to be the combination of having the person who accepts money in exchange for goods being responsible for paying the transaction cost, together with the idea of money as generalized purchasing power—*i.e.*, indifference by the household as to the identity (or home goods) of the bearer. In other words, the vendor specializes in acquiring one good—fiat money—that does not directly enter into the household's utility function. If the shopper paid the transaction cost—as we saw above in section 4.1—the utility gained from a greater variety of goods would be weighed against the cost of added variety. In contrast, in the environment in which the vendor pays the transaction costs, the vendor does not observe (or care about) any variety of goods. In the absence of acquiring goods that directly enter into the household's utility function, it is not surprising that the vendor eschews variety, trading with shoppers that minimize the total transaction costs paid by the household. If the household can sell $e-\sum_{i\in S_i'} a(i)$ for $p_t\left[e-\sum_{i\in S_i'} a(i)\right]$ on any set S_t', then the household would want to make S_t' a singleton.[14]

By inspection, it is obvious that the equilibrium outcomes for the vendor-pays case are not identical to those in the shopper-pays case when fiat money is present. In short, it matters who pays the fixed costs. In the monetary version of this economy, we have equilibria that can be radically different depending on which party to a transaction bears the cost. Moreover, the following proposition compares the welfare outcomes associated with the two monetary economies. Let $v^i(m_t;\mathbf{z}_t)$ where $i=v,s$ denote the value function computed for the vendor-pays and shopper-pays cases, respectively.

Proposition 2. *In the two monetary economies, $v^s(m_t;\mathbf{z}_t)\geq v^v(m_t;\mathbf{z}_t)$; that is, welfare in the vendor-pays environment cannot exceed welfare in the shopper-pays environment.*

Proposition 2 simply states that the lifetime utility of the representative household can never be less in the shopper-pays equilibrium than it is in the vendor-pays equilibrium. Note that the shopper could always choose to consume the good of just the next-door neighbor. So, if the household in the shopper-pays case chooses a range of goods such that $k>1$, it follows that welfare is strictly greater under the shopper-pays case than under the vendor-pays case.

The intuition is straightforward. The cost to the shopper from going to an additional location, call it the k^{th}, is twofold. First, there is the marginal

[14]With a continuum of locations, the set would vanish—*i.e.,* there would be no symmetric equilibrium when the vendor pays.

utility foregone from consuming less at each of the first $k-1$ locations so that the shopper can acquire some goods at the k^{th} location. Second, there is the marginal utility foregone because some goods are used up by transaction costs at the k^{th} location. To offset these two marginal costs, there is the marginal utility associated with quantities from the new location. As long as the marginal benefit exceeds the sum of the two marginal costs, welfare is higher. Hence, if the shopper chooses multiple locations, it follows that total welfare is greater by buying at these locations than if the shopper were to stop after trading with the first location.

Shoppers specialize in the acquiring goods while the vendor specializes in acquiring what is, in effect, an intermediate good. Money is an intermediate good used by the household to acquire final goods. Because households at different locations do not coordinate, under vendor-pays rules each household tries to maximize its acquisition of the intermediate good by minimizing its transactions costs; taking the actions of all households together, this behavior condemns all shoppers to the smallest possible choice of varieties.

4.3 Mechanisms to improve the vendor-pays equilibrium

One question immediately arises. If the shopper-pays case Pareto dominates the vendor-pays case, is there a way to re-shape the household's problem so that the dominant equilibrium of the former environment obtains in the latter? The problem is analogous to the textbook prisoner's dilemma: When the vendor pays, there is no incentive to unilaterally accept shoppers from locations more distant than the $(N-1)^{\text{st}}$ location (the location immediately next door to location 0). This is true regardless of the variety of locations open to the household's shopper. As in the prisoner's dilemma, though, a mechanism enforcing cooperation can improve the equilibrium outcome.

To illustrate this point, it is straightforward to show that there exists such a mechanism—the enforcement of symmetric trade rights—that eliminates the inefficiency of the vendor-pays setting with money. The following proposition formalizes this point.

Proposition 3. *With a costless intermediary to enforce symmetric trade rights,* $v^v(m_t; \mathbf{z}_t) = v^s(m_t; \mathbf{z}_t)$.

To prove this point, we begin by describing an environment in which an intermediary can costlessly enforce a welfare improving symmetric equilibrium.

Under the mechanism we have in mind, a household submits the choice it plans to make for the set under its control, and the intermediary dictates the set outside the household's control in a symmetric way. From the standpoint of a representative household at location 0 in the shopper-pays environment, for example, if the household submits $\{1, 2, \ldots, k\}$ as the set of locations it will pay the transaction cost to shop at, the mechanism would dictate $\{N-k, \ldots, N-2, N-1\}$ as the locations the household's vendor will accept cash from. In the vendor-pays environment, if the household submits $\{N-k', \ldots, N-2, N-1\}$ as the set of locations it will pay the transaction cost to accept cash from, the mechanism would dictate $\{1, 2, \ldots, k'\}$ as the set of locations open to the household's shopper. Households in either environment maximize utility taking the mechanism into account.

Clearly, the mechanism adds nothing to the shopper-pays environment: the household is indifferent to the locations it gets cash from, so telling it who to accept cash from imposes no constraint. The household's maximization given the mechanism is equivalent to choosing both k—locations its shopper will visit—and k'—locations that can use cash at the home location—subject to the constraint $k = k'$. That is, the household's problem, given the enforcement of symmetric trading rights, is equivalent to

$$v^s(m_t; \mathbf{z}_t) = \max_{k_t, k_t'} \left[m_t^{\alpha} k_t^{1-\alpha}\right]^{1/\alpha} + \beta v\left(\frac{p_t}{p_{t+1}}\left[e - \sum_{i=1}^{k_t} a(i)\right]; \mathbf{z}_{t+1}\right)$$
$$\text{subject to } k_t = k_t'.$$

As the reader can see, the sole difference between the problem with and without the intermediary is clear; there is an additional constraint which guarantees symmetry in trading ranges. Note, too, that k_t' does not appear in the Bellman equation for the shopper-pays case. The implication is that the constraint is costlessly satisfied in the monetary economy in which the shopper-pays the transaction cost. In other words, it is a matter of indifference to the household whether it sells to the set defined by $S_t = \{N - k_t, \ldots N-2, N-1\}$ or the set defined by $S_t' = \{N - k_t', \ldots N-2, N-1\}$ where $k_t < k_t'$. Because the shopper bears the transactions cost, the vendor's action in accumulating fiat money is costless to the representative household. Since the households are otherwise identical across locations, no shopper from farther away than k_t locations will trade with the location-0 vendor, just as no location-0 shopper will trade with a vendor farther than k_t locations away. Thus, in equilibria, $k_t = k_t'$, and the choice is the same as it was for the shopper-pays setting without an intermediary.

While the mechanism adds nothing to the shopper-pays environment, it makes a great deal of difference for the vendor-pays environment. The equilibrium that arises in that environment will now be identical to the one that obtains in the shopper-pays case. Here again the problem faced by the household can be thought of as a choice of k'—locations it incurs the transactions cost to sell to—and k—locations its shopper will visit—subject to the constraint $k = k'$. The constraint is no longer costlessly satisfied, as both k and k' enter into the household's Bellman equation:

$$v^v(m_t; \mathbf{z}_t) = \max_{k_t, k'_t} \left[m_t^\alpha k_t^{1-\alpha}\right]^{1/\alpha} + \beta v\left(\frac{p_t}{p_{t+1}}\left[e - \sum_{i=1}^{k'_t} a(i)\right]; \mathbf{z}_{t+1}\right)$$

$$\text{subject to } k'_t = k_t.$$

With the equality constraint, it is straightforward to substitute k_t for k'_t or vice versa in the Bellman equation. Thus, the problem can either be written as

$$v^v(m_t; \mathbf{z}_t) = \max_{k'_t} \left[m_t^\alpha k_t'^{1-\alpha}\right]^{1/\alpha} + \beta v\left(\frac{p_t}{p_{t+1}}\left[e - \sum_{i=1}^{k'_t} a(i)\right]; \mathbf{z}_{t+1}\right)$$

or equivalently as

$$v^v(m_t; \mathbf{z}_t) = \max_{k_t} \left[m_t^\alpha k_t^{1-\alpha}\right]^{1/\alpha} + \beta v\left(\frac{p_t}{p_{t+1}}\left[e - \sum_{i=1}^{k_t} a(i)\right]; \mathbf{z}_{t+1}\right)$$

Clearly, the maximization will yield the same choice of locations that the representative location household will accept or visit. Indeed, as one can see from the latter representation of the unconstrained Bellman equation, the vendor-pays case will yield the same outcome as the shopper-pays case. Thus, the existence of the intermediary is sufficient to eliminate the difference between the two cases.

5 Summary and conclusion

In this paper, we specify a simple general equilibrium model with differentiated consumption goods in which traders face a fixed fee to acquire goods.

The fixed fee is strictly increasing in the variety of goods consumed. To help illustrate the household decision problem, we treat each household as consisting of two individuals, each performing a specific activity. We consider two cases distinguished by which party to a transaction is responsible for bearing the fixed fee. We consider each experiment in two different payment systems: barter and money. The household objective is always to maximize lifetime welfare. Variety is desired by the household. In the barter system, the household faces a basic trade-off—to obtain greater variety, the household must pay a higher transaction cost. In the vendor-pays case, the shopper's actions are taken as given, and we solve for the equilibrium that the vendor chooses. In the shopper-pays case, the vendor's actions are taken as given and we solve for the equilibrium that the shopper chooses. In each version, the household's problem is written as if one person is choosing the locations and quantities, taking the location the other party will visit as given. This structure plays a critical role in our results. We ask whether the equilibrium is invariant to which party is responsible for the transaction cost when we analyze the problems under different payment arrangements. We focus on symmetric equilibrium.

Our key results are:

(1) In barter economies—*i.e.,* ones in which goods are exchanged for goods—the equilibria are the same whether shoppers or vendors are responsible for paying the transaction costs.

(2) This invariance fails in monetary economies—the equilibria are different in that households will consume a wider range of the differentiated products when the shopper pays the transaction costs than when the vendor pays the costs.

(3) Moreover, the two equilibria are Pareto ranked with the shopper-pays equilibrium welfare-dominating the vendor-pays equilibrium. If a third-party intermediary could enforce a symmetric trading range, the coordination failure across the two experiments would be resolved, so that $v^s(m_t; \mathbf{z}_t) = v^v(m_t; \mathbf{z}_t)$.

The differences that emerge in the two monetary economies owe chiefly to two factors. First, money is an intermediate good, and, in effect, the vendor specializes in producing the intermediate good for the household. The vendor puts no value on the variety available to other households' shoppers, and thus—in the environment where the vendor pays the transaction cost—maximizes the household's purchase of the intermediate good, money, by minimizing the range of locations the household sells the home good to. In contrast, the shopper specializes in acquiring the final consumption good. To achieve this goal, the shopper maximizes household welfare by acquiring a wider range of goods with the money available. Thus, in the environment in which shoppers bear the transaction cost, the shopper balances the marginal benefit of greater variety against the marginal transaction cost, taking the quantity of the intermediate good as given. Thus, money creates different objectives. No such distinctive objectives emerge when both parties are directly acquiring final consumption goods, which is what is happening in the barter environment.

Second, welfare is generally not the same in the two monetary economies. The two monetary equilibria are Pareto ranked, with household welfare lower in the vendor-pays environment. In that sense, the equilibrium in the vendor-pays environment has the flavor of a coordination failure. We show that an intermediary that can enforce symmetric trading rights eliminates the inefficiency associated with the vendor-pays economy. In our setup, the intermediary dictates a mechanism that avoids the kind of coordination problem that exists in the monetary economies; given the mechanism, households end up choosing the welfare-maximizing distance.

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Appendices

A Trading ranges are 'connected'

Suppose not, so a gap exists in the trading range chosen by the household. In particular, suppose that the household has chosen a trading range containing locations k and $k+2$, but not $k+1$. A gap of arbitrary length would be treated similarly.

The household's consumption vector would therefore have $c(k), c(k+2) > 0$ and $c(k+1) = 0$. Given its budget constraint, the household could remove $k+2$ from its trading range, add $k+1$, and enjoy the consumption vector

$\tilde{c}$ given by

$$\tilde{c}(k+2) = 0$$
$$\tilde{c}(k+1) = c(k+2) + a(k+2) - a(k+1)$$
$$\tilde{c}(j) = c(j) \quad (\forall j \neq k+1, k+2)$$

This produces a change in the household's utility,

$$\tilde{U} - U = [\cdots + (c(k+2) + a(k+2) - a(k+1))^{\alpha} + \cdots]^{1/\alpha} - [\cdots + c(k+2)^{\alpha} + \cdots]^{1/\alpha}. \quad \text{(A.1)}$$

$\tilde{U} - U > 0$ in (A.1), since the transaction cost function $a(\cdot)$ is increasing (and the terms represented by '$\cdots$' are identical in $\tilde{U}$ and U).

B Proof that relative prices are unity in symmetric equilibria

Consider the barter economy in which the shopper pays the transaction cost. For the other cases, the analysis is similar. Let $p(l,h)$ denote the price of good h (in units of good l) paid by a shopper from location l, and $q(l,h)$ denote the price of good h (also in units of good l) paid by a vendor at l. Suppose the household at l visits locations $l+1$ through $l+k$ and is visited by shoppers from locations $l-1$ through $l-k'$. The household at l then faces the budget constraint

$$e - \sum_{i=1}^{k} a(i) \geq \sum_{i=1}^{k} p(l, l+i)\, c(l+i) + \sum_{i=1}^{k'} q(l, l-i)\, c(l-i) \qquad \text{(B.1)}$$

The agent maximizes utility subject to this constraint (for given k and k'), which yields the following demand functions

$$c(l+i) = \left(\frac{1}{\lambda p(l, l+i)} \right)^{\frac{1}{1-\alpha}} \qquad \text{(B.2)}$$

for all $i \in \{1, 2, \ldots, k\}$ and

$$c(l-i) = \left(\frac{1}{\lambda q(l, l-i)}\right)^{\frac{1}{1-\alpha}} \tag{B.3}$$

for all $i \in \{1, 2, \ldots, k'\}$, where λ is the Lagrange multiplier on the household's budget constraint. Substituting (B.2) and (B.3) into (B.1), as an equality, gives

$$e - \sum_{j=1}^{k} a(j) = \left(\frac{1}{\lambda}\right)^{\frac{1}{1-\alpha}} \left(\sum_{j=1}^{k} p(l, l+j)^{\frac{-\alpha}{1-\alpha}} + \sum_{j=1}^{k'} q(l, l-j)^{\frac{-\alpha}{1-\alpha}}\right)$$

or

$$\left(\frac{1}{\lambda}\right)^{\frac{1}{1-\alpha}} = \frac{e - \sum_{j=1}^{k} a(j)}{\sum_{i=1}^{k} p(l, l+i)^{\frac{-\alpha}{1-\alpha}} + \sum_{i=1}^{k'} q(l, l-i)^{\frac{-\alpha}{1-\alpha}}}.$$

Thus,

$$c(l+i) = \frac{p(l, l+i)^{\frac{-1}{1-\alpha}} \left(e - \sum_{j=1}^{k} a(j)\right)}{\sum_{j=1}^{k} p(l, l+j)^{\frac{-\alpha}{1-\alpha}} + \sum_{j=1}^{k'} q(l, l-j)^{\frac{-\alpha}{1-\alpha}}}$$

and

$$c(l-i) = \frac{q(l, l-i)^{\frac{-1}{1-\alpha}} \left(e - \sum_{j=1}^{k} a(j)\right)}{\sum_{j=1}^{k} p(l, l+j)^{\frac{-\alpha}{1-\alpha}} + \sum_{j=1}^{k'} q(l, l-j)^{\frac{-\alpha}{1-\alpha}}}.$$

In a symmetric equilibrium, all households choose the same number of locations to visit (so $k = k'$), and $p(l, l+i)$ and $q(l, l-i)$ depend only on i. Consequently, we suppress the dependence on l without loss of generality, letting $p(i)$ and $q(i)$ denote these relative prices.

Since the endowment at each location (net of the transaction cost) is divided between the k visitors and the k locations visited, material balance requires:

$$e - \sum_{i=1}^{k} a(i) = \sum_{i=1}^{k} \left(\frac{q(i)^{\frac{-1}{1-\alpha}} \left(e - \sum_{j=1}^{k} a(j)\right)}{\sum_{j=1}^{k} p(j)^{\frac{-\alpha}{1-\alpha}} + \sum_{j=1}^{k} q(j)^{\frac{-\alpha}{1-\alpha}}}\right) + \sum_{i=1}^{k} \left(\frac{p(i)^{\frac{-1}{1-\alpha}} \left(e - \sum_{j=1}^{k} a(j)\right)}{\sum_{j=1}^{k} p(j)^{\frac{-\alpha}{1-\alpha}} + \sum_{j=1}^{k} q(j)^{\frac{-\alpha}{1-\alpha}}}\right)$$

or

$$\sum_{i=1}^{k} p(i)^{\frac{-\alpha}{1-\alpha}} + \sum_{i=1}^{k} q(i)^{\frac{-\alpha}{1-\alpha}} = \sum_{i=1}^{k} p(i)^{\frac{-1}{1-\alpha}} + \sum_{i=1}^{k} q(i)^{\frac{-1}{1-\alpha}}.$$

Next, note that relative prices are related by

$$q(i) = \frac{1}{p(i)}$$

—that is, in a symmetric equilibrium, the relative price paid by the vendor for a good brought from i locations away is the inverse of the relative price paid by a shopper for a good purchased i locations away. Relative prices for a symmetric equilibrium must then obey

$$\sum_{i=1}^{k} p(i)^{\frac{-\alpha}{1-\alpha}} + \sum_{i=1}^{k} p(i)^{\frac{\alpha}{1-\alpha}} = \sum_{i=1}^{k} p(i)^{\frac{-1}{1-\alpha}} + \sum_{i=1}^{k} p(i)^{\frac{1}{1-\alpha}}$$

which simplifies further to

$$\sum_{i=1}^{k} \left[\left(p(i)^{\frac{1}{1-\alpha}} + p(i)^{\frac{-1}{1-\alpha}} \right) - \left(p(i)^{\frac{\alpha}{1-\alpha}} + p(i)^{\frac{-\alpha}{1-\alpha}} \right) \right] = 0$$

It is then straightforward to show that when $\alpha \in (0,1)$, each term i in this sum is nonnegative for all $p(i) > 0$ and strictly positive for $p(i) \neq 1$. If, however, $\alpha > 1$, then each term i is nonpositive for $p(i) > 0$, and strictly negative for $p(i) \neq 1$. In either case, only $p(i) = 1$ for all i satisfies the last equation.

To verify this claim, note that each term in the sum has the form

$$z + z^{-1} - \left(z^{\alpha} + z^{-\alpha} \right),$$

where $z \equiv p(i)^{\frac{1}{1-\alpha}}$. Since $f(z) \equiv z + z^{-1}$ is convex, we have, for all positive z and x,

$$f(z) \geq f(x) + f'(x)(z - x),$$

or

$$z + z^{-1} \geq x + x^{-1} + \left(1 - x^{-2}\right)(z - x). \tag{B.4}$$

In the case of $\alpha \in (0,1)$, letting $x = z^{\alpha}$ in (B.4) yields

$$z + z^{-1} \geq \left(z^{\alpha} + z^{-\alpha}\right) + \left(1 - z^{-2\alpha}\right)(z - z^{\alpha}). \tag{B.5}$$

If $z > 1$, then $1 - z^{-2\alpha}$ and $z - z^{\alpha}$ are both positive. Alternatively, if $z < 1$, $1 - z^{-2\alpha}$ and $z - z^{\alpha}$ are both negative. Hence, the product on the right-hand side of (B.5) obeys $\left(1 - z^{-2\alpha}\right)(z - z^{\alpha}) > 0$ for $z \neq 1$. Thus, inequality (B.5) implies that $z + z^{-1} > z^{\alpha} + z^{-\alpha}$ for $z \neq 1$.

Next, consider the case in which $\alpha > 1$. Substituting $z = x^{\alpha}$ into inequality (B.4) yields

$$x^{\alpha} + x^{-\alpha} \geq x + x^{-1} + \left(1 - x^{-2}\right)(x^{\alpha} - x)$$

or

$$-\left(1 - x^{-2}\right)(x^{\alpha} - x) \geq x + x^{-1} - \left(x^{\alpha} + x^{-\alpha}\right). \tag{B.6}$$

With $\alpha > 1$, $x^{\alpha} - x$ and $1 - x^{-2}$ are both either strictly positive (if $x > 1$) or strictly negative (if $x < 1$). Thus, the left-hand side of inequality (B.6) is less than zero for any $x \neq 1$, implying that $x + x^{-1} < (x^{\alpha} + x^{-\alpha})$ for all $x \neq 1$.

Finally, for any $\alpha > 0$, it is easy to show that inequalities (B.5) and (B.6) hold as equalities when $z = x = 1$. Thus, the material balance condition for a symmetric equilibrium is satisfied if and only if the list of relative prices has $p(i) = 1$ for all i.