ELASTIC CAPITAL SUPPLY AND THE EFFECTS OF FISCAL POLICY

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Existing analyses of the effects of fiscal policy in general equilibrium models have typically been conducted under the assumption that the long-run supply of capital is perfectly elastic at a fixed rate of time preference. These analyses have shown that the long-run response of the capital stock to changes in fiscal policy is crucial to generating the potential for "multiplier" effects in these models. In this paper we ask, what are the implications of relaxing the assumption of perfectly elastic capital supply for the analysis of fiscal policy? We show that with less than perfectly elastic capital supply, the potential for multipliers is actually enhanced. (*JEL* E62, D90)

I. INTRODUCTION

Perfectly elastic or perfectly inelastic supply or demand curves have much to recommend them. Equilibrium analysis which would otherwise be fraught with ambiguity yields forth sharp predictions when one assumes either demand or supply are either perfectly elastic or inelastic. Nonetheless, this is not the way we typically teach equilibrium analysis nor, in most circumstances, perform it. Neoclassical macroeconomics is an exception to this rule. Specifically, capital accumulation models in which a representative agent maximizes the standard additively-separable, fixed-discount-factor utility function—to which class most equilibrium business cycle models based on the neoclassical growth model belong—imply a long-run supply curve for capital which is perfectly elastic at the agent's fixed rate of time preference.

This property of what we will refer to as the "standard" model is, and has been, well-known and well-criticized, even by users of the standard model.[1] But, the question of exactly where and when this assumption ceases to be innocuous—i.e., for what sorts of experiments it is or isn't a harmless simplification—has been given surprisingly short shrift.[2] In this paper we explore the implications for the equilibrium analysis of the effects of changes in government purchases of relaxing this assumption. In particular, we explore the implications of replacing the fixed discount factor β in the standard utility specification

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, l_t\right)$$

where c_t and l_t denote consumption and leisure at date t, with an endogenous discount factor, $\beta(c_t, l_t)$. In this way, discounting of future utility is allowed to depend on the agent's enjoyment of current consumption and leisure. The lifetime utility function that results from this modification is of the sort first formulated by

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1. In *partial* equilibrium, it implies an "all or nothing" type of behavioral response—when faced with constant interest rates, agents wish to hold either no capital or an infinite amount. In *general* equilibrium, unless all agents share the same common discount factor, all capital ends up in the hands of the most patient agent; when agents share the same discount factor, the long-run distribution of capital holdings across agents is indeterminate. See, for example, Becker [1980].

2. A recent exception is a paper of Gomme and Greenwood [1992], which utilizes an endogenous time preference specification similar to ours in a real business cycle model. These sorts of preferences have also, quite naturally, shown up in the open economy macro literature, where for a small open economy fixity of time preference implies an indeterminacy in the economy's long-run debt position. The need to get away from fixed rates of time preference is here very clear and has been addressed, for example, by Mendoza [1991].

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Uzawa [1968] and Epstein and Hynes [1983]. The latter of these, in particular, demonstrated through a series of examples the extent to which models with endogenous discount factors can differ substantially from their fixed-discount-factor counterparts. While these earlier papers were primarily concerned with developing theoretical results, our analysis will be primarily quantitative.

The possibility that the rate of time preference varies across individuals and over time, or equivalently, that the rate at which the future is discounted responds to the current and previous decisions of individuals, is more than a theoretical curiosum. To start with, there is a substantial body of evidence that rates of time preference differ across individuals: people are not equally patient. Much of this evidence is summarized in table 1 of Becker and Mulligan [1997]. At the aggregate level, they point to the fact that rich countries have grown slightly faster than poor countries over the past 30 years as suggesting that the residents of rich countries may have lower rates of time preference than residents of poor countries. Differences between the United States and Japan in the age-consumption profile are also consistent with differences in the rate of time preference between the United States and Japan. And the fact that income is associated with consumption growth suggests that the rich may be more patient than the poor. Detailed micro evidence on differences in rates of time preference across households is also presented by Lawrance [1991]. She finds that the rates of time preference of poor consumers are three to five percentage points higher than those of rich consumers using panel data for the United States from the PSID. Ogaki and Atkeson [1997] estimate a model in which time preference is allowed to vary across rich and poor households using household level panel data from India. Furthermore there is evidence that rates of time preference vary not just across individuals at a point in time, but also over time. Becker and Mulligan [1997] investigate the factors that may lead individuals to discount the future more or less heavily, argue that wealth causes patience, and show that the evidence supports the notion of causality from wealth to patience rather than the other way around. Also at the macro level Ogawa [1993] presents evidence that the rate of time preference varies with the stage of economic development. He finds that of the three countries whose development experience he examines, only Korea seems to exhibit a constant rate of time preference. For Japan and Taiwan he finds that the rate of time preference declines up to a certain point with the level of development, and thereafter rises. Hong [1988] presents indirect evidence of variable time preference at the macro level by looking at how the savings rate responds to the opening of trade in developing countries. In short, there is a burgeoning empirical literature documenting the existence of varying rates of time preference at both the micro and macro levels.[3]

Given that there are grounds for believing that the rate of time preference is not fixed, and is in fact partially determined by factors under the control of agents, why look at the implications for fiscal policy of relaxing the fixed time preference assumption? Recent analyses of the effects of fiscal policy in equilibrium models (see, for example, Aiyagari et al., [1992] and Baxter and King [1993]) have highlighted the importance of persistence in these shocks if they are to have conventional multiplier effects. In particular, it has been shown that capital accumulation is crucial to generating conventional multiplier effects from persistent changes in government purchases in these models. Thus there is ample reason for investigating the implications for these results of relaxing the standard assumption of a perfectly elastic long-run capital supply.

The paper is organized as follows. Section II of the paper lays out a general framework for the equilibrium analysis of shocks to government purchases, which allows for both a non-zero slope to capital's long-run supply as well as shifts of the supply schedule. The section begins with a description of the type of preferences we will use, and then sets about solving the more general model and characterizing its equilibrium in terms of efficiency conditions.

Our analysis of fiscal policy proper is undertaken from two perspectives, a "comparative steady state" analysis—exploring the two models' differing responses to truly perma-

3. For a sociobiological analysis of variable time preference, along with some supporting evidence, see Rogers [1994].

nent changes in government purchases—and a quantitative, numerical analysis of the effects of both transitory and persistent changes on the models' complete dynamical systems.

The steady state analysis is useful for developing the intuition of what makes models with flexible time preference "different." In particular, under reasonable parameter values, introducing flexible time preference in a manner consistent with an upward-sloping long-run capital supply curve can generate much *larger* output effects—"multipliers"—than the standard model, while keeping the employment effect basically the same. Also, permanent changes in government purchases, even when financed through lump-sum taxes, give rise to long-run interest rate effects in the more general model. Steady-state consumption may actually *rise* in response to a permanent increase in purchases, depending on the responsiveness of time preference to changes in consumption and leisure. The representative agent is, nonetheless, worse off as a result.

Section IV contains our analysis of the effects of both transitory and persistent changes in government purchases on output, employment, investment and so forth in both the short and long runs. We approximate the models' dynamics in a linear fashion and report responses of the approximate dynamical systems to deviations in purchases which display different degrees of persistence. We find that the results of Baxter and King [1993] and Aiyagari et al., [1992]—that transitory shocks to purchases yield smaller output effects than persistent shocks—continue to obtain even in our more general framework. However, in the case of transitory shocks, we find that the impact effects on employment, consumption and output are much larger, and the impact effect on investment much smaller, in the flexible-time preference model than in the fixed-time preference model. We also find that in the case of transitory shocks, the propagation is significantly weaker in the model with flexible time preference: in the wake of shocks to government purchases the transition back to the steady state is quite rapid.[4] The same is true for the responses of the real wage and the real interest rate. In the case of persistent—in fact "nearly permanent"—shocks, the effects at impact on all quantity and price variables are qualitatively the same across the two models, but much larger in the more general (flexible-time preference) model. Subsequent to impact, the differing responses of the two models is accounted for largely by a "capital accumulation effect," present under flexible time preference, which we discuss in our steady state analysis.

4. This suggests that flexible time preference will not help resolve the "propagation problem" that characterizes standard general equilibrium models of business cycle.

II. THE MODEL

Except for the endogeneity of the rate of time preference, our model is the standard neoclassical growth model, augmented to incorporate government purchases, which has been analyzed by King [1989], Baxter and King [1993] and Aiyagari et al., [1992].

Output at each date is produced from capital, k_t, and labor hours, n_t, according to a concave, constant-returns production function F. Output is divided between consumption, c_t, gross investment, i_t, and government purchases of goods, g_t:

$$F(k_t, n_t) \equiv y_t \geq c_t + i_t + g_t \tag{1}$$

The economy's capital stock evolves according to

$$k_{t+1} = (1-\delta)k_t + i_t \tag{2}$$

where δ is the depreciation rate of capital. Available hours of effort are constrained by $0 \leq n_t \leq 1$, where we normalize the time endowment to unity.

The preferences we employ specify lifetime expected utility at date zero as

$$U_0 = E_0\left\{\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t-1}\beta\left(c_s, l_s\right)\right)u\left(c_t, l_t\right)\right\}, \tag{3}$$

where $l_t = 1 - n_t$ denotes hours of leisure. The key feature of equation (3) is the discount factor which the agent applies between periods t and $t+1$, $\beta(c_t, l_t)$, which is a function of consumption and leisure in period t. This form follows Uzawa [1968], Epstein and Hynes [1983] and Epstein [1983] and has been used in real business cycle models by Mendoza [1991]; it differs from the time-additive case only in the dependence of the discount factor on current consumption and leisure. Were $\beta(c_t,l_t)$ simply a constant β, lifetime utility would obey the usual

$$(4) \qquad U_0 = E_0\left\{\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)\right\}.$$

Out of the more general class of "recursive utility functions," the utility specification embodied in equation (3) has the advantage of tractability from a computational standpoint, as well as consistency with the expected utility hypothesis.[5]

Following Mendoza [1991], we treat momentary utility u and the discount factor β as depending not on the levels c and l directly, but rather on the level of a "composite commodity" $h(c, l)$. That is, we specify:

$$u(c, l) = v[h(c, l)]$$

and

$$\beta(c, l) = \theta[h(c, l)]$$

where $h: R_+ \times [0,1] \rightarrow R_+$ is an increasing function which aggregates consumption and leisure into the composite good $h(c,l)$. The functions $v: R_+ \rightarrow R$ and $\theta: R_+ \rightarrow [0,1]$ then associate levels of momentary utility and discounting with quantities of the composite good $h(c,l)$. We assume, naturally, that v is also increasing, so that higher levels of the composite good are associated with higher levels of momentary utility $v[h(c,l)]$. As we discuss below, we assume primarily for stability reasons that $\theta'(h) < 0$, so that an increase in the agent's consumption of the composite good $h(c,l)$ results in greater discounting of future "installments" of momentary utility. In terms of $\beta(c,l)$—since $h(c,l)$ is increasing in c and l—an increase in today's consumption or leisure, *ceteris paribus*, leads to a smaller discount factor applied to future utility. Below, and in the Appendix, we discuss in more detail the specific functional forms we adopt for h, v and θ, and the parameter restrictions we impose to guarantee stability of the economy's dynamics.

The market structure is competitive. The representative agent rents labor services and capital to firms at competitively determined prices. Income from labor and capital is used to finance purchases of consumption and investment, and to pay a lump-sum tax to the government. Government tax revenues are used to finance purchases of output, which we treat as simply being thrown away.[6] We assume that government purchases are financed through lump-sum, rather than distortionary, taxes in order to focus solely on the effects of government purchases as a pure drain on output.[7] Also, under lump-sum financing, optima and equilibria will coincide under standard assumptions, so we may treat the equilibrium as the solution to a social planning problem—maximizing utility (3) subject to (1) and (2), as well as the usual nonnegativity constraints, given a stochastic process for government purchases. Once optimal allocations are calculated, prices can be found by examining the appropriate marginal rates of substitution or transformation.

We now proceed to describe the solution to the social planning problem. Let

$$\pi_t \equiv v'[h(c_t, l_t)] + \theta'[h(c_t, l_t)]E_t(V_{t+1}),$$

where $E_t(V_{t+1})$ denotes the expected value, as of date t, of maximized lifetime utility from date t+1 onward. The variable π_t summarizes

5. "Recursive" utility functions—of which (3) and the standard specification (4) are representatives—have the feature that lifetime utility from today on can be written as function of today's consumption (or consumption and leisure) and lifetime utility from tomorrow on. It is this feature which makes dynamic programming possible with such preferences. Within this class—as Epstein [1983] as shown—only the standard specification and (3) are consistent with expected utility.

6. It would be straightforward to allow government purchases to enhance the utility or production possibilities of the representative agent, but doing so would only complicate the analysis without adding much of substance.

7. Baxter and King [1993], however, have shown that the presence of distortionary taxation has important implications within the standard model, and this would no doubt be true in our model as well. The presence of distortionary taxes also renders important the question of financing.

the welfare consequences of an increment to the composite good $h(c,l)$ at date t. The term $v'[h(c_t,l_t)]$ denotes the immediate gain in momentary utility from an increment to the composite commodity, while the term $\theta'[h(c_t,l_t)]$ $E_t(V_{t+1})$ summarizes the effect on discounted future utility. Note that in the absence of flexible time preference, this second term is always equal to zero.

As we show in the Appendix, the solution to the planning problem is then described by the following efficiency conditions and constraints. The first is the standard "intratemporal" first-order condition

$$\frac{h_2(c_t,1-n_t)}{h_1(c_t,1-n_t)} = F_2(k_t, n_t), \tag{5}$$

which equates the agent's within-period marginal rate of substitution between leisure and consumption to the marginal product of labor. The "intertemporal" conditions characterizing the solution are the Euler equation, which for this problem takes the form

$$h_1(c_t, 1-n_t)\pi_t = \theta[h(c_t, 1-n_t)] \times \tag{6}$$

$$E_t\left[h_1(c_{t+1}, 1-n_{t+1})\pi_{t+1}\left(F_1(k_{t+1}, n_{t+1}) + 1 - \delta\right)\right],$$

the law of motion for the capital stock,

$$k_{t+1} = F(k_t, n_t) + (1-\delta)k_t - g_t - c_t, \tag{7}$$

and the exogenous process for government purchases. The first and last of the preceding three equations—(5) and (7)—are standard for models of this sort. The contribution of the analysis below all hinges on the middle equation (6), the intertemporal efficiency condition, so it is worthwhile spending a little time fleshing out an intuitive interpretation of this condition. The left hand side of this expression can be thought of as the cost in utility terms of foregoing a unit of consumption at date t. This cost has two components: the reduction in the amount of the composite commodity available to the consumer, $h_1(c_t, 1-n_t)$, and the effect that this decline has on current utility and the discounted value of future utility, π_t. Absent adjustment costs for capital, a unit reduction in consumption today means that there is one more unit of capital available for productive purposes at date t+1, increasing the availability of consumption goods at date t+1 by the marginal product of capital, $F_1(k_{t+1}, n_{t+1})$, plus the undepreciated portion of the capital stock, $1-\delta$. To translate the extra availability of consumption goods into units of the composite commodity, simply multiply by $h_1(c_{t+1}, 1-n_{t+1})$, which can in turn be converted to utility units by multiplying by π_{t+1}.

The efficiency conditions above form the basis for our subsequent analyses and quantitative experiments. We focus first on the long-run effects of government purchases.

III. LONG-RUN OUTPUT EFFECTS

To get a feel for the impact of flexible time preference, it's worth initially considering the deterministic steady state of the model. The deterministic steady state has government purchases, consumption, hours and the stock of capital constant. The intratemporal first-order condition (5) becomes

$$\frac{h_2(c,l-n)}{h_1(c,1-n)} = F_2(k, n), \tag{8}$$

while the terms $h_1(c_t,1-n_t)\pi_t$ and $h_1(c_{t+1},1-n_{t+1})$ π_{t+1} on either side of the Euler equation (6) drop out to yield

$$1 = \theta[h(c,1-n)](F_1(k, n) + 1-\delta). \tag{9}$$

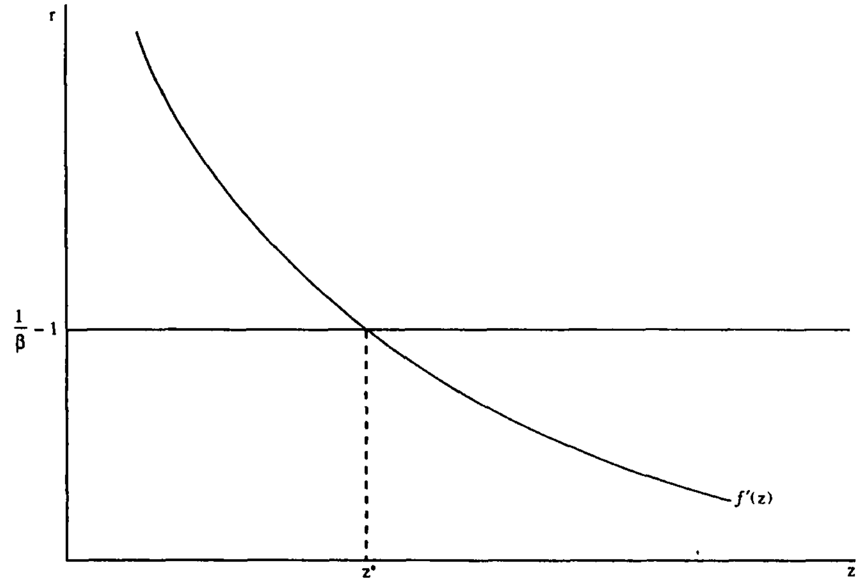
In the steady state, investment is equal to depreciated capital, and hence the resource constraint (7) reduces to

$$F(k, n) - \delta k = c + g. \tag{10}$$

Fixed time preference

Suppose that time preference is fixed—so that $\theta[h(c,1-n)]$ is equal to a constant, say β. Given that and the degree-one homogeneity of F, the capital-labor ratio will be determined by (9), independent of g. Changes in g consequently have no effect on either the steady state real interest rate or the steady state real wage. Consequently, the right hand side of (8)

FIGURE 1
Long-Run Capital Market, Standard Model



is fixed as well—in essence steady-state labor *demand* is rendered perfectly elastic at a fixed real wage, independent of g as well.

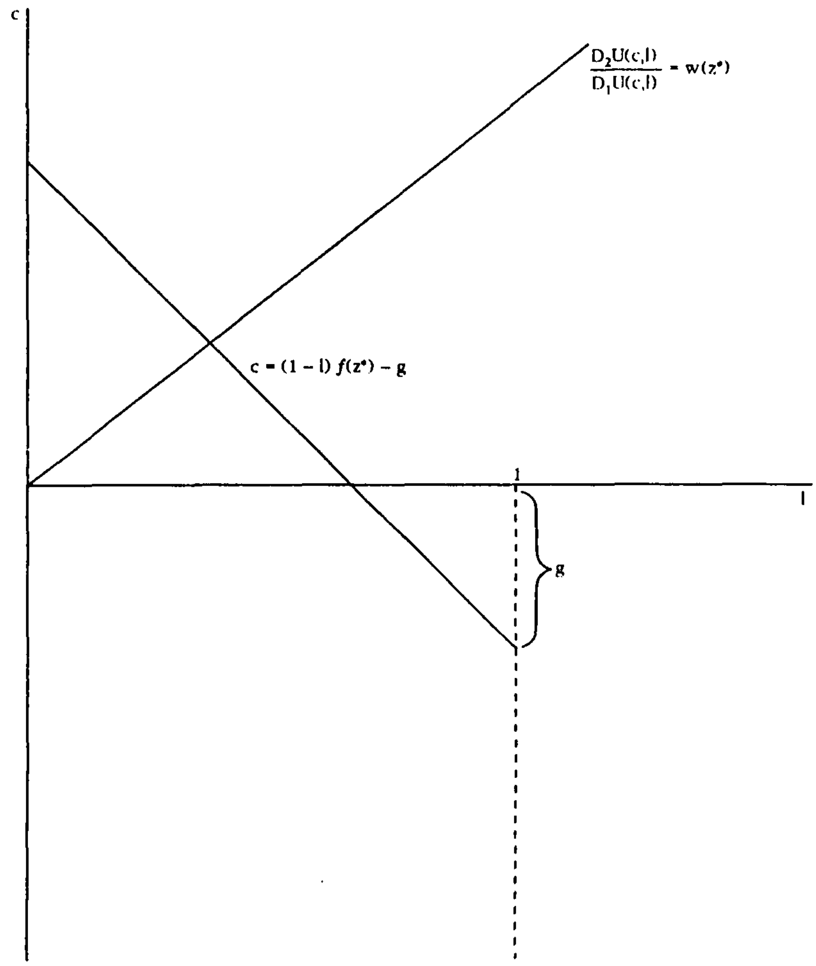
Let $z = k/n$, and let z^* denote the capital-labor ratio determined by the capital market clearing condition (9), with $\theta[h(c,1-n)] = \beta$. The long-run effect of a change in g boils down to calculating the derivative of the function $n(g)$ defined implicitly by

$$\frac{h_2[(F(z^*,1)-\delta z^*)n(g)-g,1-n(g)]}{h_1[(F(z^*,1)-\delta z^*)n(g)-g,1-n(g)]} = F_2(z^*,1),$$

which uses the fact that steady state consumption satisfies $c = (F(k/n,1) - \delta k/n)n - g = (F(z,1) - \delta z)n - g$. A true long-run "multiplier"—a greater than one-for-one response of output to a change in the level of government purchases—will exist whenever $n'(g) > 1/F(z^*,1)$, since steady state output is $nF(z^*,1)$. It is straightforward to show that there are specifications of g, F, β, δ, and h—equivalently, in this case, u—which yield this result. Given that we have relatively more confidence, empirically, in what the first four primitives on this list should look like than we do in regard to u, the existence proposition would typically be stated as "a long-run output multiplier will exist if leisure is sufficiently income-elastic."

What's going on here can be visualized in a pair of simple graphs. Figure 1 shows long-run equilibrium in the "capital market"—the determination of the capital-labor ratio by (9) under fixed time preference. Figure 2 then illustrates the determination of steady state consumption and leisure. Given that the capital-labor ratio has been determined in the capital market, the long-run equilibrium occurs at the intersection of two curves in consumption-leisure space. One curve is simply the "income expansion path" of $h(c,l)$ when the wage rate is given by $w(z^*) = F_2(z^*,1)$—i.e., the collection of all pairs (c,l) satisfying the intratemporal efficiency condition $h_2(c,l)/h_1(c,l) = w(z^*)$. The other curve represents the locus of *feasible* consumption-leisure pairs given the capital-labor ratio z^*. It is the downward-sloping straight line determined by the equation $c = (1-l)[F(z^*,1) - \delta z^*] - g$. Permanent changes in g induce parallel shifts in this "budget line," and the magnitude of the resulting changes in leisure—equivalently, labor—de-

FIGURE 2
Long-Run Consumption-Leisure Choice, Standard Model



pend on the slopes of the "income expansion path" and "budget line" near the equilibrium. (See Figure 3)

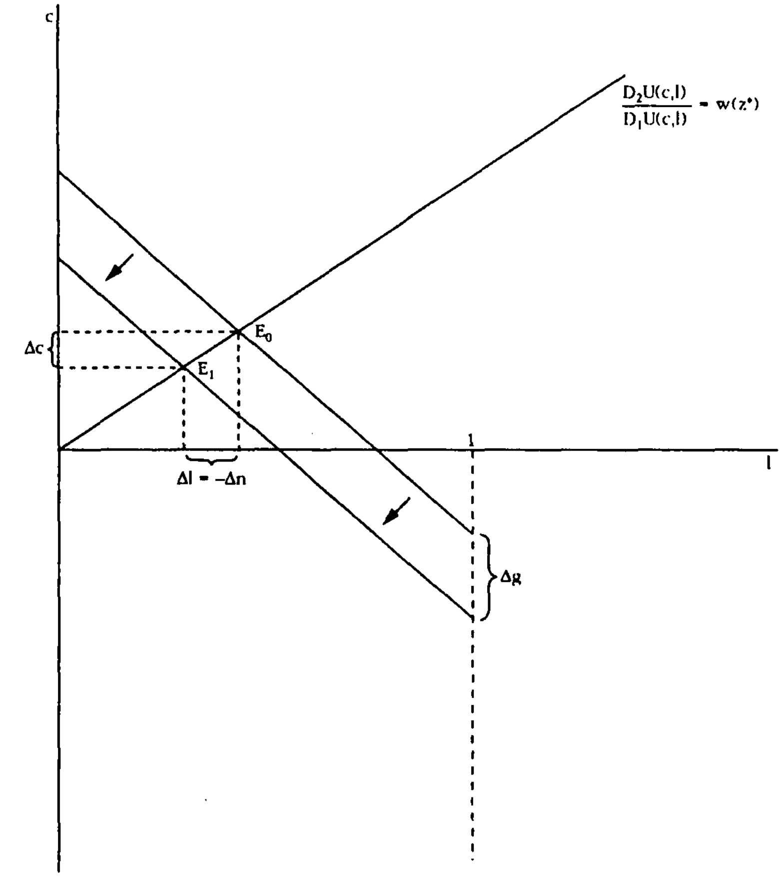
Since the capital-labor ratio is fixed, any change in n is implicitly accompanied by an equal-proportioned change in k. Also, changes in output are proportional to changes in labor as well, and steady-state consumption clearly falls. Obviously, in such a model, permanent—i.e., steady-state—changes in g have no interest rate effects nor real wage effects.

The natural experiment to conduct in this framework would be to demonstrate—given accepted parametrizations of F, β and g— exactly *how* "income-elastic" leisure has to be in order to generate a given long-run response of output to government expenditures. In percentage terms, since a one percent change in leisure yields a $-(1 - n)/n$ percent change in labor, a moderate responsiveness of leisure can yield a large responsiveness of labor.[8] One would then ask whether the set of numbers which are "sufficient" overlap with the set of numbers which are "plausible".[9]

8. Following King, *op. cit.*, estimates of the long-run fraction of discretionary hours devoted to labor range variously from two-tenths to one-third, implying $(1 - n)/n$ in the range of two to four.

9. Taking what is "plausible," for example, from estimates such as those surveyed in Pencavel [1986].

FIGURE 3
Long-Run Effect of $\Delta g > 0$, Standard Model



Endogenous Time Preference

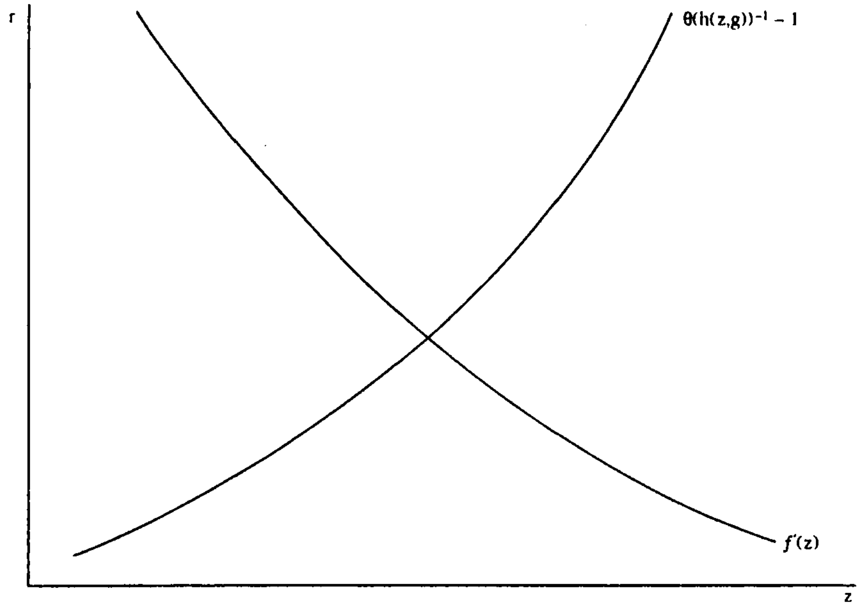
Now, consider what happens when time preference is endogenous. Again, let z denote the capital-labor ratio. For a given value of z, the intratemporal efficiency condition (8) again defines an "income expansion path" consisting of pairs (c,l) such that $h_2(c,l)/h_1(c,l) = F_2(z,1)$. The resource constraint again determines a downward-sloping straight line given by $c = (1 - l)\ [F(z,1) - \delta z] - g$. The intersection of the two curves yields choices of consumption and leisure given z and g—call them $c(z,g)$ and $l(z,g)$ (just as in Figure 2). In a slight abuse of notation, let $h(z,g) \equiv h[c(z,g),l(z,g)]$—that is, $h(z,g)$ is the value of the composite good $h(c,l)$ consistent with the resource constraint and intratemporal efficiency *given* values for both z and g. The equilibrium value of z in the flexible-discount-factor case is then determined by the capital-market clearing condition

$$\frac{1}{\theta[h(z,g)]} = F_1(z,1) + 1 - \delta.$$

The solution to this equation—assuming one exists—will give the steady state capital-labor ratio as a function of g—$z(g)$, say (see Figure 4). Going back to the "intratemporal" picture yields $c(g) \equiv c(z(g),g)$ and $l(g) \equiv l(z(g),g)$.

Now, one can show under standard assumptions that $h(z,g)$ is increasing in z, for a given value of g, and decreasing in g, for a given value of z. If we assume that $\theta'(h) < 0$, $1/\theta[h(z,g)]$ defines an upward-sloping long-run supply curve for capital—actually for z—in the space with z on the horizontal axis and the real interest rate on the vertical axis. Given

FIGURE 4
Long-Run Capital Market, Flexible Time Preference



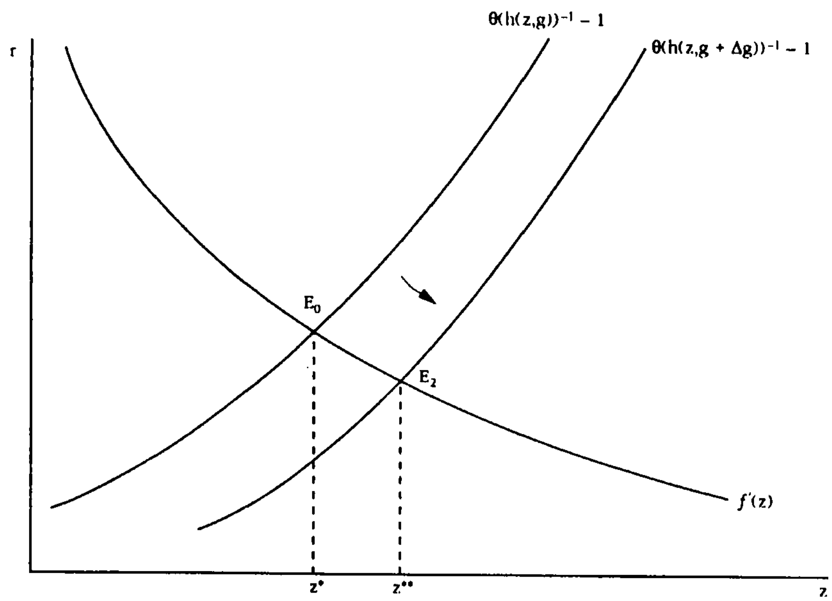
what we've said about the dependence of $h(z,g)$ on g, and θ on h, the supply schedule will shift out—i.e., down and to the right—in response to an increase in g (see Figure 5).

Now, suppose the economy is in a steady state, given a constant level of purchases g. A permanent increase in purchases from g to $g + \Delta g$, say, will impact simultaneously on the steady-state values of c, n and z. Heuristically, though, it's instructive to view the change in the equilibrium through "partial equilibrium" glasses—and in terms of our two diagrams characterizing the consumption-leisure choice given the capital-labor ratio and the long-run capital market. Given the original steady-state value of z, an increase in government spending impacts on the consumption-leisure choice by shifting downward in parallel fashion the "budget line" in the consumption-leisure diagram—just as in the fixed-discount-factor model (see Figure 6). This has the effect of lowering consumption and leisure—i.e., increasing labor—and, consequently, lowering the steady-state flow of the composite good $h(c,l)$.

In the fixed-discount-factor case, this would be the end of the story, but here the change in $h(c,l)$ impacts on discounting and hence the capital market. The long-run supply of capital shifts out, leading to a lower steady-state interest rate and a higher capital-labor ratio (again as in Figure 5). The increased capital-labor ratio in turn impacts on the consumption-leisure choice, affecting both the "income expansion path"—rotating it upward, as the real wage increases with a higher capital-labor ratio—and the "budget line"—increasing its slope and vertical intercept. The contribution of this second adjustment is clearly positive with respect to consumption—that is, relative to the initial "fixed-z" movement—and ambiguous with respect to leisure. Allowing the capital-labor ratio to adjust can mean either more or less leisure taken in the steady state, relative to the initial fixed-z effect. If we think of the fixed-z effect as the new steady state of the fixed-discount-factor model, then allowing for a flexible discount factor implies an employment effect which can be greater than, less than, or equal to the fixed-discount-factor employment effect.

Suppose the shifts in the "budget line" and "income expansion path" engendered by the

FIGURE 5
Effect on Long-Run Capital Market of $\Delta g > 0$, Flexible Time Preference



increase in the capital-labor ratio z lead to a new steady state with roughly the same level of employment as was the case when z was held fixed. Is it then the case that the steady-state *output* effect should be the same in either case? The answer is no, since when the capital-labor ratio changes, movements in output are no longer proportional to movements in labor hours—and here, recalling the outward shift of capital's long-run supply, we have an increase in the capital-labor ratio. Thus, even when introducing flexibility of the discount factor engenders no difference in steady state *employment* effects, effects on *output* are always magnified, relative to the fixed-discount-factor case, by the accumulation of additional steady-state capital.

This scenario is, roughly speaking, exactly what plays itself out when the model is evaluated numerically, given standard parameter values. Precisely, given values for things like factor income shares, expenditure shares, the steady-state interest rate and parameters of h at an original steady state, the changes in c, n and z in response to a small change in g can be written as functions of a parameter to which the slope of capital's long run supply is proportional. For a wide range of values for this parameter the employment effect of a given change in steady state g varies slightly, in fact falling, while the output effect increases rather dramatically as the parameter moves further away from zero, which corresponds to the fixed-discount-factor case. The enhanced output effects are due almost entirely to increases in the capital-labor ratio.

To be concrete, we assume that the composite good $h(c,l)$ has the Cobb-Douglas form

$$h(c,l) = c^{1-\psi}\, l^{\psi},$$

while the function v mapping values of $h(c,l)$ into levels of momentary utility is given by

$$v(h) = \frac{h^{1-\sigma} - 1}{1 - \sigma},$$

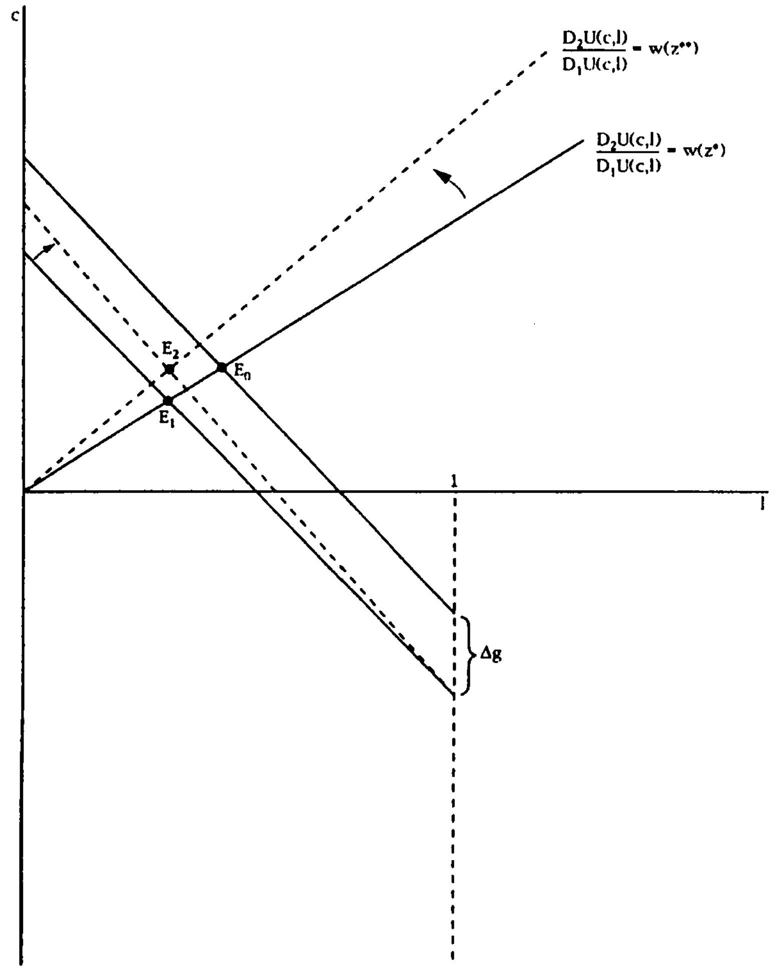
for $\sigma > 0$. Thus, momentary utility has the standard form

$$u(c,l) = \frac{(c^{l-\psi}\, l^{\psi})^{1-\sigma} - 1}{1 - \sigma},$$

for $\sigma \neq 1$ and

$$u(c,l) = (1 - \psi)\ln(c) + \psi \ln(l),$$

FIGURE 6
Impact of Higher z on Long-Run Consumption-Leisure Choice



for $\sigma = 1$. In standard fashion, we will assume also that the production function takes the Cobb-Douglas form $F(k,n) = k^{1-\alpha}n^{\alpha}$.

We take no stand on the exact functional form of $\theta(h)$. Rather, since our quantitative analyses here and below rely on linearization techniques, solutions depend only on the elasticities of $\theta(h)$—$h\theta'(h)/\theta(h)$ and $h\theta''(h)/\theta'(h)$—which we will subsequently denote by η_1 and η_2, respectively. The fixed-discount-factor case can then be recovered by setting both of these parameters equal to zero. The results we obtain—and in fact the stability of the dynamic system—when η_1 and η_2 are non-zero will depend on both the sizes and signs of these parameters. The Appendix discusses stability restrictions on these and other parameters, though at this point it's worthwhile to discuss at least one important choice which we make—the sign of $\theta'(h)$.

Since the composite commodity $h(c,l)$ is increasing in consumption and leisure, which are in turn increasing in wealth, signing $\theta'(h)$ is tantamount to asking the well-worn question—dating back to Fisher [1930] and Hayek [1941]—as to whether impatience increases or decreases with wealth. The case of $\theta' > 0$—so that increases in within-period consumption or leisure bring the discount factor closer to one—can be thought of as reflecting the idea that the more happiness one receives today, the more "patient" one becomes with respect to future happiness. Conversely, $\theta' < 0$ corresponds to the equally arguable notion that the more happiness one receives today, the less one cares about future installments of happiness.[10] Perhaps the most compelling case—offered originally by Hayek, subsequently formalized by Epstein [1983], Lucas and Stokey [1984] and others—is that $\theta' < 0$ guarantees long-run stability in the one-sector model. This is most easily seen by abstracting for a moment from labor supply. If labor supply were inelastic, then one could think of $1/\theta[h(f(k) - \delta k - g)]$ as representing the long-run supply curve for capital. Then, at least for values of k with $f(k) - \delta k$ increasing, $\theta' < 0$ corresponds to an upward-sloping long-run supply curve. This, in fact, is the assumption we maintain throughout our analysis.

So much for the theoretical arguments for the appropriate choice of value for θ'. Can we bring any empirical evidence to bear on this question? Lawrance [1991], using PSID data, finds evidence that subjective discount factors *rise* with labor income, though it's not clear what implication this has for our assumption of $\theta' < 0$. In particular, individual rates of time preference in her specification are assumed to be independent of individual consumption. The Euler equations which she uses to obtain her estimates are thus identical to the ones the standard model would generate, except in that the discount factors are allowed to differ across individuals. Further, as a little algebra applied to the impulses responses we later report will show, the discount factor and labor income *are* positively related in the experiments we conduct as well.

Taking account of our functional form assumptions, differentiation of the capital-market clearing condition (9) about the steady state yields

$$\eta_1\left((1-\psi)\hat{c} - \psi\frac{n}{1-n}\hat{n}\right)$$

$$- \alpha(1 - \beta(1-\delta))\,\hat{z} = 0,$$

where a hat over a variable denotes percentage deviation from the initial steady state, and β denotes the (initial) steady-state value of the endogenous discount factor, $\theta[h(c,l)]$. Note that when η_1 equals zero, the expression reduces to $\hat{z} = 0$, which reflects the fact, mentioned earlier, that the capital-labor ratio is fixed in the long run in the constant-discount-factor case.

The elasticity η_1 enters only into the capital-market equation, the steady-state versions of the other equations—the intratemporal efficiency condition and the resource constraint—being standard. Once we specify values for α, δ, ψ, g and the initial steady-state value of the discount factor β (which can always be set independently of η_1 and η_2), we can derive solutions for $\hat{c}/\hat{g}$, $\hat{n}/\hat{g}$ and so forth as functions of η_1. Setting $\eta_1 = 0$ recovers the fixed discount factor case. Given solutions for $\hat{z}/\hat{g}$ and $\hat{n}/\hat{g}$, one can also obtain expressions for $\hat{y}/\hat{g}$, which is simply $\hat{n}/\hat{g} + (1-\alpha)\hat{z}/\hat{g}$, and the "multiplier" dy/dg, which is simply $(y/g)\hat{y}/\hat{g}$.

Following standard procedure—and in order to maintain comparability with other results—we set $\alpha = .58$ and, following Baxter and King [1993], $\beta = .95$. The parameter ψ is set, given the other parameter values, so that $n = .20$ is chosen by the agent in the steady state.[11] We choose the empirically plausible value of .20 for government's steady-state share of national output (g/y). The depreciation rate, δ, is set equal to 5.0%, which implies a steady-state share of investment in aggregate output of 20.5% (and a steady state share of consumption of 59.5%).[12]

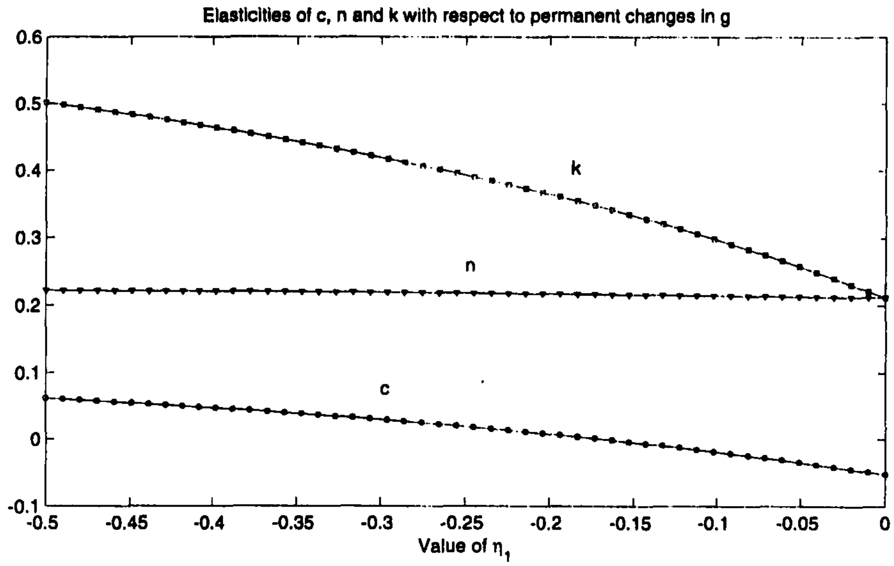
Figures 7 illustrates the consequences of allowing η_1 to vary between 0 and –0.5. Panel A of Figure 7 shows how the elasticities of consumption, labor and capital with respect to changes in the level of government purchases change as we allow for less elasticity in the long-run supply of capital. Starting at the rightmost point on the graph we see that when $\eta_1 = 0$, i.e., when the rate of time preference is fixed, the elasticity of consumption with

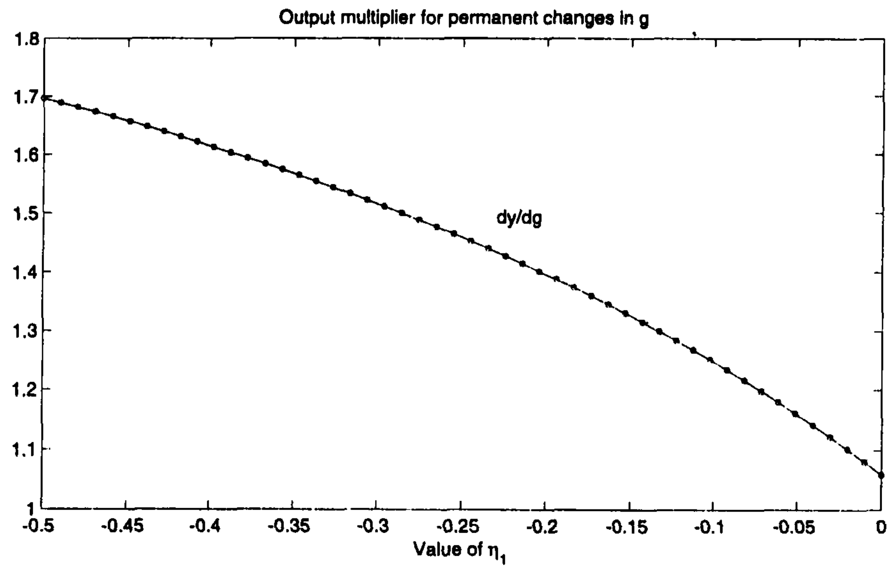
10. This is to be understood in a marginal sense—the contribution to today's lifetime utility of a small increment to future utility is smaller the higher are current consumption and leisure.

11. From the intratemporal efficiency condition we have the restriction that $\psi = ((\alpha/s_c)(1-n)/n)/(1 + ((\alpha/s_c)(1-n)/n))$ where $s_c = 1 - \delta(k/y) - s_g$ is the share of consumption in steady-state output.

12. Hercowitz [1986], using Canadian national accounts data, obtained an estimate of δ of about 5% per annum.

FIGURE 7
Long-Run Elasticities and Output Multipliers for Various Values of η_1





respect to changes in government purchases is negative, while the responses of capital and employment are the same (implying that the capital-labor ratio is constant). Allowing η_1 to take on values less than zero does not lead to any significant change in the response of effort to changes in government purchases, but it does lead to a greater response of the capital stock. As the value of η_1 gets smaller, we see that consumption may actually rise in response to steady-state increases in government purchases.

IV. THE EFFECTS OF TRANSITORY AND PERSISTENT CHANGES IN GOVERNMENT PURCHASES

The next stage in our analysis is to compare dynamic response of an economy with an elastic long-run supply of capital to changes in the level of government purchases. We do this by looking at the impulse responses of the full dynamical system to an increase in government spending under various assumptions as to the persistence of the disturbance, given assumptions about the time preference parameters η_1 and η_2.[13]

The process for government spending is assumed, in percentage deviations from steady state, to follow an AR(1) process, with autocorrelation parameter ρ. We examine the effects of a shock to government purchases under three different assumptions about its persistence. The first is a purely temporary shock with $\rho = 0$. The second is a permanent shock, which is mimicked by setting ρ arbitrarily close to one (we set $\rho = .9999$). The third is an intermediate case with $\rho = .94$, which is the estimated value reported by Burnside et al., [1993]. All impulse responses are for a 1% shock to g, and plot the corresponding paths of $\hat{c}$, $\hat{n}$, $\hat{k}$, etc. The horizontal scales in all cases are in years. In all three cases we assume that $\eta_1 = -.4$ and $\eta_2 = -.45$. These choices are admittedly somewhat arbitrary, but presumably give us some sense of how the assumption of less-than-perfectly-elastic capital supply affects the analysis. Experiments with different values for these parameters suggest that our results are reasonably robust.

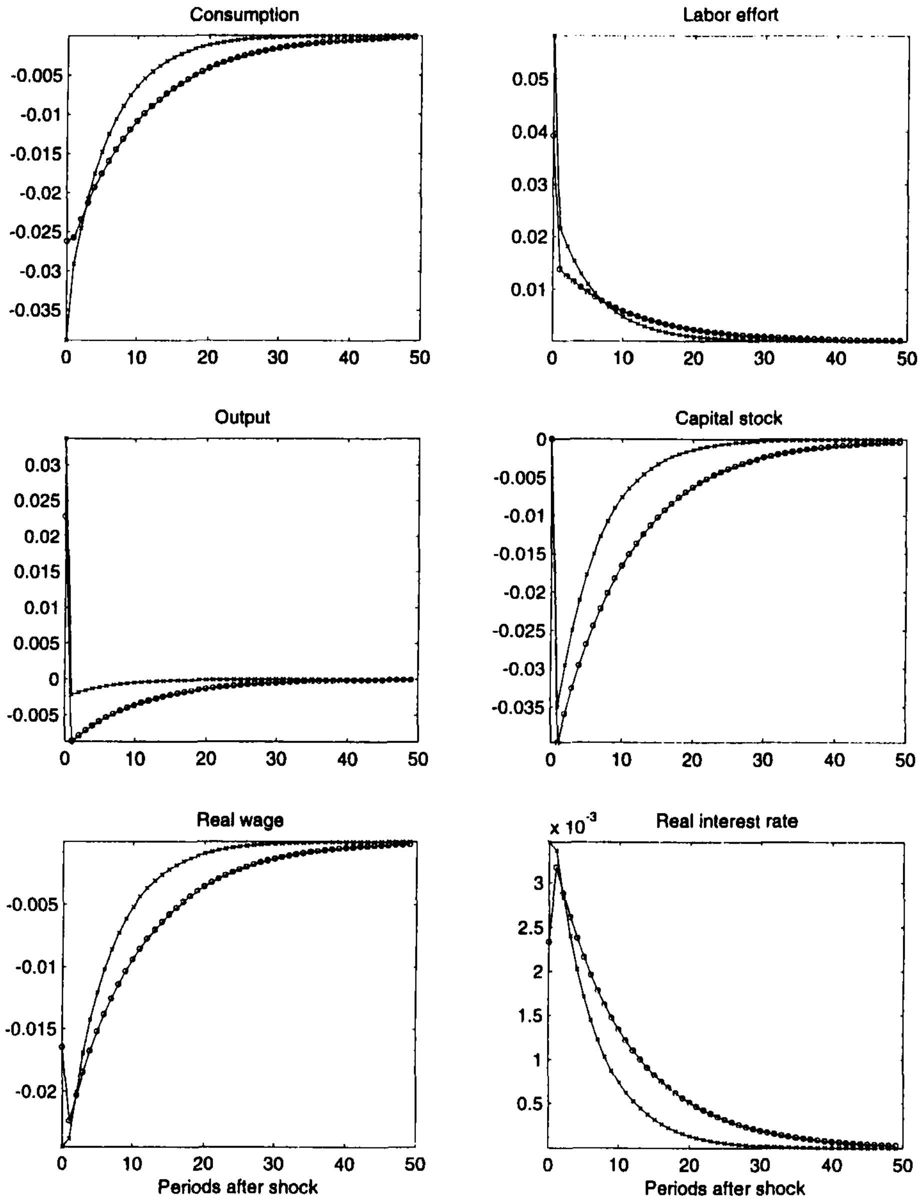
The first set of six pictures, Figure 8, records the responses of consumption, effort, the capital stock, output, the interest rate and the real wage for the case of a purely transitory ($\rho = 0$) shock to purchases, under $\sigma = 1.5$. The two paths in the picture of each variable are that variable's response under flexible time preference—in all cases the "x" line—and fixed time preference—the "o" line. The main features one observes in these responses are that, first of all, flexible time preference of the sort we have specified yields a *qualitatively* similar response for four of the six variables as obtains in the fixed time preference case, with the real wage and interest rate being slight exceptions in their transitions back to steady state. At impact, in both cases consumption and investment (not shown) fall, while effort, and hence—because capital is initially fixed—output, rise. The real wage falls at impact, and the real interest rate rises. In the second and subsequent periods, capital in both cases is below its steady state level, owing to the smaller investment in the impact period. At this point, the transitional dynamics of both models dictate that effort and investment should be high, and consumption low, relative to their steady states until the systems converge back to their original positions. The paths of the real wage and real interest rate differ in that, in the fixed-time-preference case, each variable moves further from its steady state in period two, and then monotonically returns back, yielding a modest "hump-shaped" path for each variable. In the flexible-time-preference case, each of the two variables begins its transition back to steady state immediately.

Quantitatively, the flexible-time-preference responses show much larger effects at impact on consumption, effort and output than fixed time preference responses. The same can be said for the at-impact responses of the real interest rate and the real wage. Accordingly, the response at impact of investment is smaller in the flexible case, and in the subsequent period the capital stock is nearer to its steady-state value than under fixed time preference. Since the model's transitional dynamics from an initially low capital stock take over at this point, and since capital is not quite so far out of line with its steady-state value, the flexible-time-preference responses show much less propagation of the shock than do the fixed-time-preference responses.

The greater at-impact responses of consumption and effort—as well as the smaller response of investment—have a simple diagrammatic explanation in terms of the consumption-leisure-investment choice which the representative agent faces at impact. *Given* the level of investment optimal *prior* to the shock, the transitory increase in g has the effect of a parallel shift down in today's consumption-leisure possibilities set. Consumption decreases and labor effort increases. *But*, the originally optimal level of investment is

13. Our solution method, which is employs the linearization techniques developed by King, Plosser and Rebelo [1988], is described in the Appendix.

FIGURE 8
Effect of a Temporary ($\rho = 0$) Change in g



no longer optimal. If we view investment as chosen to equate its marginal cost—the marginal utility of consumption—with its marginal benefit—the discounted expected marginal value of capital—then we've had an upward shift in the marginal cost schedule. In the fixed-discount-factor case, that's all that occurs—consequently, investment is reduced somewhat from its previously optimal level, and the initial negative effects on consumption and leisure checked somewhat. But, with flexible time preference, the increase in the marginal cost of investment is accompanied by an increase in its *marginal benefit*—since the expected marginal value of capital is discounted less as today's consumption of the composite good $h(c,l)$ falls. Consequently, the adjustment in investment is smaller—so investment falls by less in the flexible-time-preference case—and hence the "correction" of the initial effects on consumption and effort lessened.

The next set of six pictures—Figure 9—shows, for the same variables and parameter values, responses to a "permanent" ($\rho = .9999$) shock to purchases. As one would expect, for both flexible and fixed time preference, the effects at impact on consumption, effort and output are much larger now—for example, under fixed time preference, the impact multiplier on output is about .85 in the permanent case versus about .1 in the purely transitory case.[14] There is also now a positive effect on investment, as the increase in the marginal cost of investment is accompanied now in both fixed and flexible cases by a large increase in the marginal benefit of investment—if the shock is going to be around for awhile, the marginal value of extra capital for those periods is high. With flexible time preference, however, we again get a substantial added boost on the marginal benefit side due to the change in discounting. Consequently, the at-impact responses of all variables are larger under flexible time preference than under fixed time preference. This difference is particularly noticeable in investment—where the difference is by more than a factor of five—and in output—where the impact multiplier is now nearly 1.5.

After impact, the dynamics reflect the transitions of the variable to their "new steady states." As our comparative steady state analysis showed, the difference between fixed and flexible time preference in this regard is dominated by the desire to greatly increase steady-state capital.

The paths of the interest rate and real wage under flexible time preference are precisely what one would expect given the movements in labor effort and capital—after large impact effects, both quickly settle to their new steady states, the interest rate lower, the real wage higher.

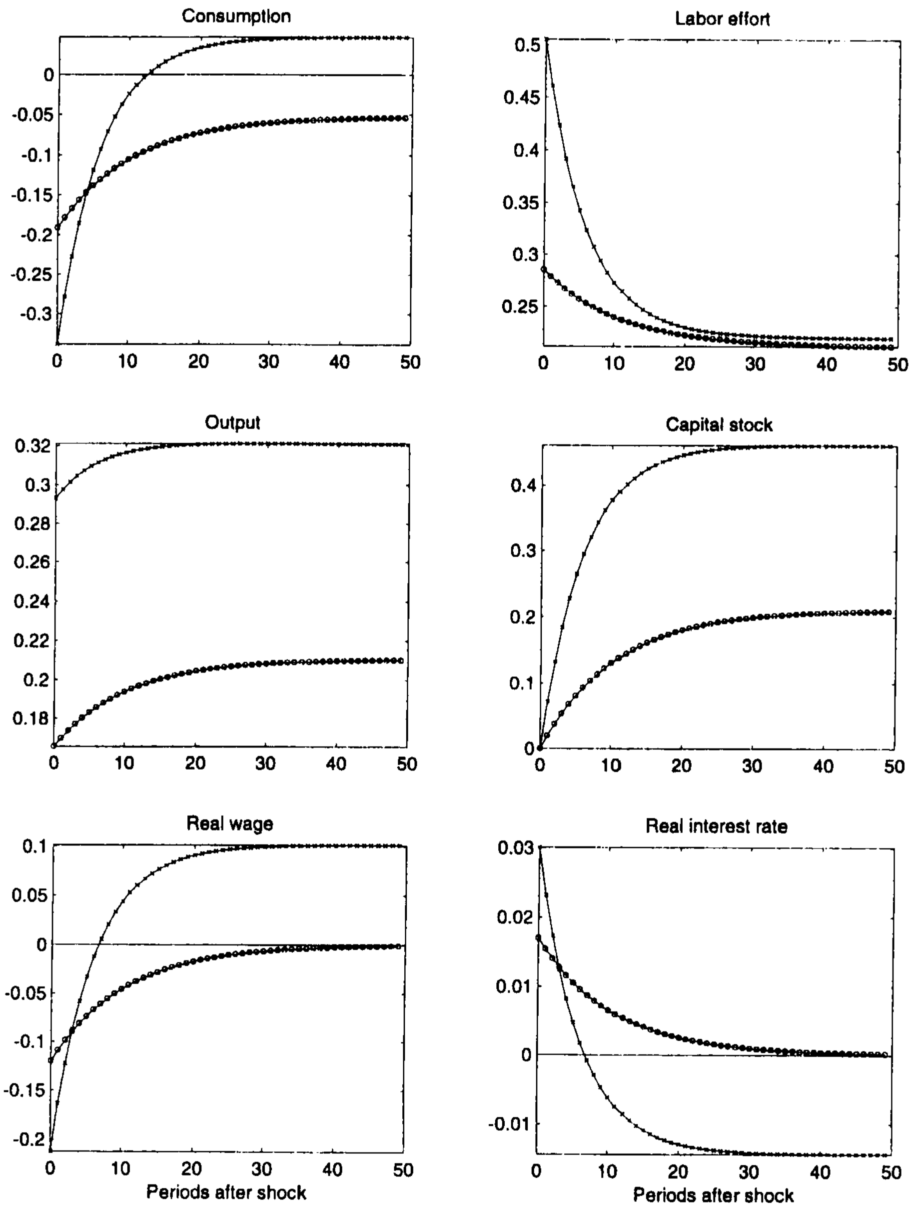
The third set of pictures—Figure 10—illustrate the effect of a shock to purchases when the persistence parameter is chosen to match postwar United States data, $\rho = .94$, as estimated by Burnside et al., [1993]. The responses of the key aggregates are now dramatically different depending on whether the rate of time preference is fixed or flexible. Starting with the response of consumption, note that consumption falls by more in the flexible time preference case, but recovers its steady state level much more rapidly. This is possible because of the persistently greater response of output following the innovation to government spending, which is in turn primarily attributable to the response of capital. Effort also increases by more in the fixed time preference case than in the flexible time preference case, but it is the qualitative difference in the response of capital in each case that plays the key role in the response of output, as well as the responses of the real wage and interest rate. Under flexible time preference, households accumulate capital at a much more rapid rate to smooth out the effect of the shock to government purchases.

V. CONCLUSIONS

The manner in which the spending decisions of governments affect the aggregate economy is one of the central questions in macroeconomics. In this paper we have extended the existing literature on the equilibrium approach to fiscal policy to allow for endogenous time preference, thereby generating an upward-sloping long-run supply curve for capital. This contrasts with the existing analyses which assume a perfectly elastic long-run supply curve for capital at the representative agent's rate of time preference. We showed that generalizing the analysis in this manner enhances the output effects of persistent changes in government purchases. The reason for this is the enhanced effect on capital accumulation of permanent changes in wealth. Our results also show that it is possible for steady state consumption to increase in response to a permanent increase in government purchases. This is in direct contrast to the standard model with fixed time preference, where consumption must always fall in

14. Recall that with $s_g = .20$, the multiplier dy/dg is five times the elasticity $\hat{y}/\hat{g}$. Since $\hat{g}_1 = 1$ in our experiments, the impact multipliers are five times the values of $\hat{y}_1$ recorded in Figures 7.4 and 8.4.

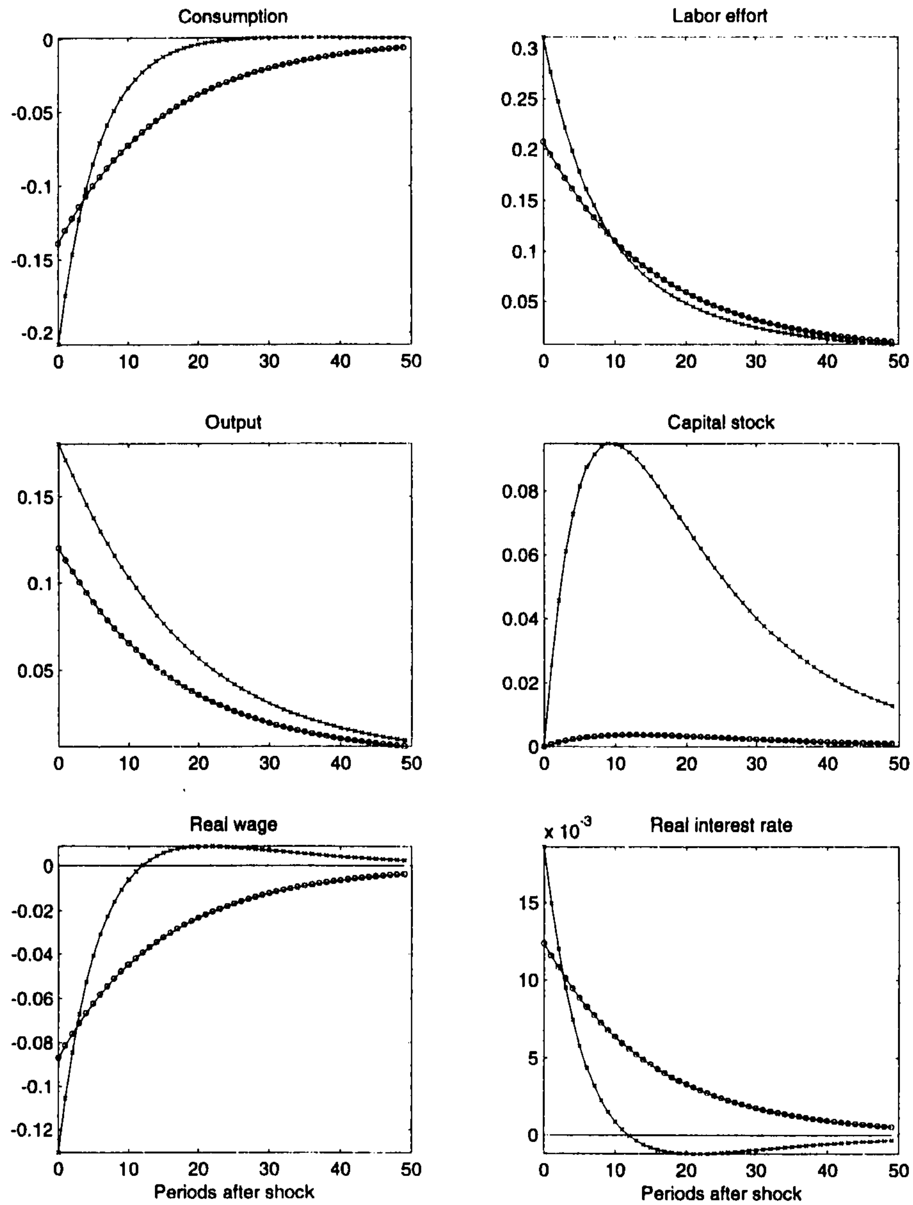
FIGURE 9
Effect of a "permanent" ($\rho = .9999$) Change in g



response to a permanent increase in government purchases.

In this paper we have limited our analysis of fiscal policy to effects of government consumption, financed in a non-distortionary manner. An obvious (and straightforward) direction for future research is to examine the effects of additional fiscal instrument, in particular distortionary taxes on labor and capital, along the lines of Baxter and King [1993]. This is a potentially interesting avenue for research, given that one of the most important consequences of a flexible rate of time preference is the possibility that the long-run in-

FIGURE 10
Effect of a Persistent ($\rho = .94$) Change in g



cidence of factor income taxes is borne by both capital and labor. This is in sharp contrast to the standard, fixed-time-preference framework, in which labor alone bears the long-burden of factor income taxation. Such an extension would come closer to being in fact a quantitative implementation of the ideas in Epstein and Hynes [1983].

A number of other areas for future research also suggest themselves. In particular, our model suggests directions for empirical work, such as testing for the presence of interest rate effects or real wage effects in response to permanent changes in government purchases—effects which obtain under endogenous time preference but not under fixed time preference. The presence or absence of such effects can potentially provide a crucial test of the endogenous time preference formulation.

Finally, it is important to be clear about what is sacrificed in moving to a model with endogenous time preference. In relaxing the assumption of a fixed discount factor, there are many directions one could move in. What's more, in relaxing the fixity of time preference, one faces "trade-offs" along several dimensions. First of all, recursivity and stationarity—implying time-consistency and amenability to dynamic programming—need not necessarily be maintained, though the tractability afforded by recursive, stationary preferences is costly to forego. Likewise, should the preferences be consistent with the expected utility hypothesis? Numerous arguments have been made for moving away from the von Neumann-Morgenstern framework—for example, Epstein and Zin [1991], Farmer [1990] and Weil [1990], to cite but a few. In the interest of deviating as little as possible from the standard model, so as not to cloud our conclusions in a multiplicity of alterations, we opted to maintain consistency with expected utility. Finally, should preferences be consistent with non-stochastic balanced growth? This is a feature of the standard model, when momentary utility is taken to be homogeneous of a fixed degree or logarithmically homogeneous in consumption. We would like to preserve this feature, but as one can see from inspection of Epstein's form for expected-utility-consistent stationary, recursive preferences, this will only be possible if the discount factor is fixed.[15] Apparently, the only intersection of these sets of preferences—stationary and recursive, consistent with expected utility and consistent with balanced growth—is the standard time-additive utility function, with homogeneous or logarithmic momentary utility.[16]

15. In order to be consistent with balanced growth, the intertemporal marginal rate of substitution in consumption must be independent of the scale of consumption.

16. This conjecture is based on results in Duffie and Epstein [1992], Epstein [1983] and Dolmas [1996].

APPENDIX

Deriving the Efficiency Conditions

The Bellman equation for the social planning problem is

$$V(k, g) = \max_{c,n} u(c,1-n) + \beta(c,1-n)E[V(k', g'): g],$$

subject to $k' = F(k,n) + (1-\delta)k - g - c$ and the various nonnegativity constraints. The conditional expectation operator derives from the assumed law of motion for government purchases—

$$\ln g_t = \rho \ln g_{t-1} + (1-\rho)\ln g + \xi_t.$$

If one substitutes $F(k,n) + (1-\delta)k - g - c$ for k' on the right-hand side, and takes the first-order conditions with respect to c and n, one obtains

$$u_1(c,1-n) + \beta_1(c,1-n)E[V(k', g'): g] = \beta(c,1-n)E[V_1(k', g'): g]$$

and

$$u_2(c,1-n) + \beta_2(c,1-n)E[V(k', g'): g] = \beta(c,1-n)E[V_1(k', g'): g] \times F_2(k, n),$$

for c and n, respectively. Further, under our assumptions that $u(c,l) = v[h(c,l)]$ and $\beta(c,l) = \theta[h(c,l)]$, the two first order conditions reduce to

$$h_1(c,1-n)\pi = \beta(c,1-n)E[V_1(k', g'): g]$$

and

$$h_2(c,1-n)\pi = \beta(c,1-n)E[V_1(k', g'): g] \times F_2(k, n),$$

where

$$\pi = v'[h(c,1-n)] + \theta'[h(c,1-n)]E[V(k', g'): g].$$

Combining these two conditions yields the intratemporal first-order condition:

$$\frac{h_2(c,1-n)}{h_1(c,1-n)} = F_2(k, n).$$

The Euler equation is obtained in standard fashion by applying the envelope theorem to the right-hand side of Bellman's equation in order to obtain an expression for $V_1(k,g)$. Substitute $F(k,n) + (1-\delta)k - g - k'$ for c on the right-hand side of Bellman's equation, and suppose that n and k' are being chosen optimally. Differentiation with respect to k gives

$$V_1(k,g) = \{u_1(c,1-n) + \beta_1(c,1-n)E[V(k',g'):g]\} \times \{F_1(k,n)+1-\delta\} = h_1(c,1-n)\pi \times \{F_1(k,n)+1-\delta\}.$$

Substituting this expression for V_1 into the right-hand side of the first-order condition for consumption then yields the Euler equation (6) given in section II.

Numerical Solution Technique

The impulse responses reported in section IV were calculated using a straightforward extension of the linearization methods used by King, Plosser and Rebelo in their [1988a] and described in detail in [1988b]. This approach involves expressing the efficiency conditions and constraints which characterize equilibrium in a form which is, in essence, the discrete-time analogue to an optimal control system—i.e., in terms of control variables, state variables and costate variables. The efficiency conditions and constraints are then linearized around the system's deterministic steady state. Linearization of the system's intratemporal efficiency conditions gives a linear feedback rule determining the controls—now in their percentage deviations from their steady state values—in terms of the states and costates, also in percentage deviation form. The linearized intertemporal conditions—e.g., Euler equations and laws of motion for the capital stock and exogenous variables such as government purchases and technology shocks—together with the linear feedback rule for the controls, implies a difference equation system in the states and costates alone. This difference equation is then solved in standard fashion—"stable roots backwards and unstable roots forward"—to give the evolution of the costates and endogenous state variables in terms of initial and terminal conditions and the path of the exogenous state variables. Initial conditions for the costate variables are implied by the requirement that the system converge to a steady state, the linear-approximation analogue to a transversality condition. The interested reader should consult King, Plosser and Rebelo [1988b] for more details of this procedure; in what follows, we will simply show how our framework maps into the King-Plosser-Rebelo framework.[17]

In our case, we begin with the conditions derived from the dynamic programming solution to the planner's problem, detailed in the previous section. We then adapt this set of efficiency conditions to the King-Plosser-Rebelo framework by introducing two new variables, one a costate variable and the other a "costate-like" variable. In particular, let λ_t denote the date-t expected discounted marginal value of next-period's capital—i.e.,

$$\lambda_t = \theta[h(c_t, 1-n_t)]E_t[V_1(k_{t+1}, g_{t+1})]$$

—and let μ_t denote date-t expected maximized value of lifetime utility from period t+1 on—i.e.,

$$\mu_t = E_t[V(k_{t+1}, g_{t+1})].$$

With these definitions, the conditions described in the previous section may be written as follows. The intratemporal efficiency conditions become

$$v'(h_t)h_1(c_t, 1-n_t) + \theta'(h_t)h_1(c_t,1-n_t)\mu_t = \lambda_t$$

and

$$v'(h_t)h_2(c_t,1-n_t) + \theta'(h_t)h_2(c_t,1-n_t)\,\mu_t = \lambda_t F_2(k_t, n_t),$$

where $h_t \equiv h(c_t, 1-n_t)$. These equations link the value of the "control vector" (c_t,n_t) to the "state" and "costate" variables k_t, λ_t and μ_t. While government purchases do not enter directly in the intratemporal conditions, the full "state-costate vector" at each date t would include the exogenous state variable g_t. Linearizing these two equations around the economy's deterministic steady state gives a linear feedback rule of the form $(\hat{c}, \hat{n})' = M(\hat{k}, \hat{\lambda}, \hat{\mu}, \hat{g})'$, where "hats" over variables denote percentage deviations from steady state values, and M is a matrix depending on the economy's deep parameters.

As for the intertemporal efficiency conditions, the Euler equation becomes, upon substitution of λ_t for $h_1(c_t,l_t)\pi_t$,

$$\lambda_t = \theta(h_t) E_t[\lambda_{t+1}(F_1(k_{t+1}, n_{t+1}) + 1 - \delta)].$$

The Bellman equation implies the following law of motion for μ:

$$\mu_t = E_t[v(h_{t+1}) + \theta(h_{t+1})\mu_{t+1}].$$

The equations which complete the dynamical system are the law of motion for the exogenous state variable g_t,

$$\ln g_{t+1} = \rho \ln g_t + (1-\rho)l\,g + \xi_{t+1},$$

and the economy's resource constraint, which gives a law of motion for the capital stock:

$$k_{t+1} = F(k_t, n_t) + (1-\delta)k_t - c_t - g_t.$$

In the King-Plosser-Rebelo methodology, the four intertemporal equations are linearized around the economy's deterministic steady state, under the assumption of perfect foresight—i.e., dropping expectations and treating next-period random variables as known realizations.

17. The MATLAB programs which we used to solve the model are available on request from the authors.

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